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# LINEAR AND NONLINEAR FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS: AN APPLICATION OF COLLOCATION METHOD

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Abstract. This work examines the collocation approach used to solve linear and nonlinear Fredholm integro- differential equations numerically. To convert the problem into an algebraic system of equations, standard collocation points are used. The algebraic equations were then solved using the matrix inversion approach. The method's uniqueness was established, and its efficiency, accuracy, and consistency were demonstrated through the solution of numerical problems.

## 1. Introduction

Mathematical equations known as "Integro-Differential Equations" (IDEs) combine integrals and derivatives. They appear in a variety of scientific and engineering domains, such as physics, biology, economics, and finance, where systems display memory- or history-dependent behavior. Integro-Differential Equations (IDEs) incorporate the influence of previous values of the unknown function through the integration term, in contrast to Ordinary Differential Equations (ODEs), which solely involve derivatives. Problems involving diffusion, wave propagation, population dynamics, control theory, and other topics frequently include integral-differential equations. They offer a more accurate explanation of events that show memory effects or spatial interactions. Since the majority of IDEs cannot be resolved analytically, research has concentrated on creating numerical techniques to achieve approximations of solutions [28] .Many methods for determining the numerical solution of integro-differential equations, include the Adomian decompositions method by [15, 19], Collocation method by [2, 3, 8, 20], Hybrid linear multistep method [17, 18],

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Chebyshev-Galerkin method [13], Bernoulli matrix method [10], Differential transform method [12], Lagrange Interpolation [27], Bernstein Polynomials Method [14], Differential transformation [11], Chebyshev polynomials [16], Weighted mean-value theorem [1], Optimal Auxiliary Function Method (OAFM) [29], Block pulse functions operational matrices [26] and Spectral Homotopy Analysis Method [6].A nonstandard finite difference method for numerically solving linear Fredholm integro-differential equations was presented by [21]. The Fredholm integro-differential equation is transformed into a system of non-linear algebraic equations using the nonstandard finite difference method and the repeated/composite trapezoidal quadrature method. [4] solved first-order Volterra integro-differential equations using the standard collocation method. Furthermore, in their research, [8, 22, 23, 24] utilized the collocation method to solve integro-differential equations, whereas [25] employed least squares collocation for fractional integro-differential equations. The class of integro-differential equations was restated in terms of the generated polynomial, assuming an approximate solution. We obtained a system of linear algebraic equations after solving for the unknown by collocating the resulting equation at several locations within the range [0, 1]. A collocation approach for the computational solution of the Fredholm-Volterra fractional order of integro-differential equations was presented by [5]. After obtaining the problem in linear integral form, they used typical collocation points to translate it into a set of linear algebraic equations. In this study, we consider linear and nonlinear Fredholm integro-differential equation of the form

$$y^{(\alpha)}(x) = g(x) + \int_0^1 K(x,t)\,(y(t))^m\,dt \;\; 0 \le x,t \le 1, \alpha = 1, m \ge 1 \tag{1}$$

subject to initial condition

$$y^{(j)}(0) = q_j,\;\; j = 0, 1, \ldots, N \tag{2}$$

where $K(x,t)$ is the Fredholm integral kernel function, $g(x)$ is the known function, and $y(x)$ is an unknown function to be determined.

## 2. Basic definitions

We give certain definitions and fundamental notions in this section for the purpose of problem formulation.

**Definition 2.1.** [4] *Let $(a_m), m \ge 0$ be a sequence of real numbers. The power series in $k$ with coefficients $a_n$ is an expression.*

$$y(k) = \sum_{n=0}^{N} a_n k^n \tag{3}$$

**Definition 2.2.** [4] *The desired collocation points within an interval are determined using this method. i.e. [a,b] and is provided by*

$$k_i = a + \frac{(b-a)i}{M}, i = 1, 2, 3, ...M \tag{4}$$

**Definition 2.3.** [4] *Let $z(s)$ be an integrable function, then*

$${}_0I_x^{\alpha}\left(z(s)\right) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} z(s)ds \tag{5}$$

**Definition 2.4.** [4] *Let $y(x)$ be a continuous function, then*

$$ {}_0I_x^{\beta}\left(y^{(\beta)}y(x)\right) = y(x) - \sum_{k=0}^{N} \frac{y^{(k)}(0)}{k!}x^k \tag{6}$$

**Definition 2.5.** [4] *A metric on a set $M$ is a function $d : M \times M \longrightarrow \mathbb{R}$ with the following properties. For all $x, y \in M$ (a) $d(x, y) \geq 0$; (b) $d(x, y) = 0 \iff x = y$ (c) $d(x, y) = d(y, x)$ (d) $d(x, y) \leq d(x, z) + d(x, y)$ If $d$ is a metric on $M$, then the pair $(M, d)$ is called a metric space.*

**Definition 2.6.** *The Beta function can be defined in terms of the Gamma function as*

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x + y)},\ x, y \in R^+$$

**Definition 2.7.** *Let $(X, d)$ be a metric space, A mapping $T : X \longrightarrow X$ is Lipschitzian if $\exists$ a constant $L > 0$ such that $d(Tx, Ty) \leq Ld(x, y)\ \forall\ x, y \in X$.*

## 3. Methodology

Here, we implement collocation approach for the numerical solution of nonlinear fredholm integro- differential equations.

**Theorem (3.0)** (Banach Contraction Principle) [7]: Let $(X, d)$ be a complete

metric space, then each contraction mapping $T : X \to X$ has a unique fixed point $x$ of $T$ in $X$, such that $T(x) = x$

**Lemma (3.1)** (Integral form): Let $y\epsilon\mathbb{C}\left((0, 1), \mathbb{R}\right)$ be the solution to equation (1) with equation(2), then it is equivalent to

$$y(x) = U(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x - s)^{\alpha-1} \left( \int_0^1 K(x, t) \left(y(t)\right)^m dt \right) ds \tag{7}$$

where

$$U(x) = \sum_{k=0}^{N} \frac{y^{(k)}(0)}{k!}x^k + \frac{1}{\Gamma(\alpha)} \int_0^x (x - s)^{\alpha-1} g(s)ds$$

*Proof.* Multiply equation (1) by ${}_0I_x^{\alpha}(.)$ gives

$$ {}_0I_x^{\alpha}\left(y^{(\alpha)}(x)\right) = {}_0I_x^{\alpha}\left(g(x)\right) + {}_0I_x^{\alpha}\left( \int_0^1 K(x, t) \left(y(t)\right)^m dt \right)$$

using equation (6) gives

$$y(x) = \sum_{k=0}^{N} \frac{y^{(k)}(0)}{k!}x^k + {}_0I_x^{\alpha}\left(g(x)\right) + {}_0I_x^{\alpha}\left( \int_0^1 K(x, t) \left(y(t)\right)^m dt \right)$$

using equation (5) gives

$$y(x) = \sum_{k=0}^{N} \frac{y^{(k)}(0)}{k!}x^k + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left(g(x)\right) ds + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left( \int_0^1 K(x, t) \left(y(t)\right)^m dt \right) ds$$

$$y(x) = U(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x - s)^{\alpha-1} \left( \int_0^1 K(x, t) \left(y(t)\right)^m dt \right) ds$$

where

$$U(x) = \sum_{k=0}^{N} \frac{y^{(k)}(0)}{k!} x^k + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} g(s) ds$$

□

3.1. **Method of Solution.** We approximate the solution of equation (7) by the polynomial approximate solution in the form

$$y(x) = \sum_{n=0}^{N} a_n x^n \tag{8}$$

where $a_n$ is the coefficients to be determined. hence,

$$y^m(x) = \sum_{n=m}^{N} a_n \frac{\Gamma(n+1)}{\Gamma(n-m+1)} x^{n-m}, n \geq m \tag{9}$$

substituting equation (8) into equation (7) gives

$$\sum_{n=0}^{N} a_n x^n = U(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left( \int_0^1 K(x,t) \left( \sum_{n=0}^{N} a_n t^n \right)^m dt \right) ds \tag{10}$$

where

$$U(x) = \sum_{k=0}^{N} \frac{y^{(k)}(0)}{k!} x^k + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} g(s) ds$$

substituting $K(s,t) = t^i s^j$

$$\sum_{n=0}^{N} a_n x^n = U(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left( \sum_{n=0}^{N} a_n \int_0^1 s^i t^{nm+j} dt \right) ds$$

$$\sum_{n=0}^{N} a_n x^n = U(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left( \sum_{n=0}^{N} a_n s^i \int_0^1 t^{nm+j} dt \right) ds$$

$$\sum_{n=0}^{N} a_n x^n = U(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left( \sum_{n=0}^{N} a_n s^i \left[ \frac{t^{nm+j+1}}{nm+j+1} \right]_0^1 \right) ds$$

$$\sum_{n=0}^{N} a_n x^n = U(x) + \sum_{n=0}^{N} a_n \frac{1^{nm+j+1}}{(nm+j+1)} \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \, s^i ds \tag{11}$$

Let $x - s = (1-u)x$,then $s = ux \implies ds = xdu$, substitute into equation (11) gives

$$\sum_{n=0}^{N} a_n x^n = U(x) + \sum_{n=0}^{N} a_n \frac{1^{nm+j+1}}{(nm+j+1)} \frac{1}{\Gamma(\alpha)} \int_0^1 ((1-u)\,x)^{\alpha-1} (ux)^i x \; du$$

using definition 1.6, gives

$$\sum_{n=0}^{N} a_n x^n = U(x) + \sum_{n=0}^{N} a_n \frac{\Gamma(i+1) 1^{n-m+j+1}}{(nm+j+1)\,\Gamma(\alpha+i+1)} x^{\alpha+i} \tag{12}$$

collocating equation (12) at $x_i$

$$\sum_{n=0}^{N} a_n x_i^n = U(x_i) + \sum_{n=0}^{N} a_n \frac{\Gamma(i+1) 1^{n-m+j+1}}{(nm+j+1)\,\Gamma(\alpha+i+1)} x_i^{\alpha+i} \tag{13}$$

Equation (13) is the algebraic equation that is solved to obtain the values of $a_n$ and substituted into the approximate solution to give the numerical solution.

## 4. Uniqueness of the Method

In this section, we establish the uniqueness of the method by introducing the following theorem and hypothesis: $H_1$ : Let $T : X \to X$ be a mapping for $y_1, y_2 \in X$,There exist a constant, $L > 0$. such that

$$|y_1^m(t) - y_2^m(t)| \leq L\,|y_1(t) - y_2(t)|$$

$H_2$ : There exist a function $K^* \in C\left([0,1] \times [0,1], \mathbb{R}\right)$, the set of all positive functions such that

$$K^* = \max_{x\in[0,1]} \int_0^1 |K(x,t)|\,dt < \infty$$

$H_3$ : The function $g \in \mathbb{R}$ is continuous. **Theorem 4.1:** Assume the $H_1$- $H_3$ hold. If

$$\left(\frac{LK^*}{\Gamma\left(\alpha+1\right)}\right) < 1 \tag{14}$$

then there exist a unique solution $y(x) \in T$

*Proof.* Let $y_1(x),\ y_2(x) \in X$,then

$$\left(Ty_1\right)(x) = U(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left(\int_0^1 K(x,t)\left(y_1^m(t)\right)dt\right) ds \tag{15}$$

and

$$\left(Ty_2\right)(x) = U(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left(\int_0^1 K(x,t)\left(y_2^m(t)\right)dt\right) ds \tag{16}$$

substract equation (16) from equation (15) gives

$$\left(Ty_1\right)(x) - \left(Ty_2\right)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left(\int_0^1 K(s,t)\left[y_1^m(t) - y_2^m(t)\right]dt\right) ds$$

taking the absolute value gives

$$\left|\left(Ty_1\right)(x) - \left(Ty_2\right)(x)\right| \leq \frac{1}{\Gamma(\alpha)} \int_0^x (x-s)^{\alpha-1} \left(\int_0^1 |K(s,t)|\,|y_1^m(t) - y_2^m(t)|\,dt\right) ds$$

taking maximum of both sides and using $H_1$ *and* $H_2$

$$d\left(Ty_1(x), Ty_2(x)\right) \leq \left[\frac{LK^*}{\Gamma(\alpha+1)}\right] d(y_1, y_2)$$

based on the inequality (14) we have

$$d\left(Ty_1(x), Ty_2(x)\right) \leq d(y_1, y_2)$$

By the Banach contraction principle, we can conclude that T has a unique fixed point. □

## 5. Convergence of the method

**Theorem 5.1: (**Convergence of method) Let $(X, d)$ be a metric space and $T : X \longrightarrow X$ be a continuous mapping and $y_N(x), y_{N-1}(x)\epsilon X$ are approximate solutions of equation (7). Let $\Delta_N(x) = |y_N(x) - y_{N-1}(x)|$ , if $\lim_{N\to 0}(\Delta_N(x)) \to 0$, then the method converges to exact solution.

*Proof.* Let $y_1(x),\ y_2(x)$ be approximated by $y_N(x) = \sum_{n=0}^{M} a_n x^n = \ \phi(x)\ \mathbf{A}$ *and* $y_{N-1} = \sum_{n=0}^{M} b_n x^n = \ \phi(x)\ \mathbf{B}$ respectively. Substitute the approximate solution into equation (7) gives

$$Ty_N(x) = U(x) + \ \frac{1}{\Gamma(\alpha)}\int_0^x (x-s)^{\alpha-1}\left(\int_0^1 K(x,t)\,(\ \phi(t)^m)\,dt\right) ds\mathbf{A}$$

Similarly

$$Ty_{N-1}(x) = U(x) + \ \frac{1}{\Gamma(\alpha)}\int_0^x (x-s)^{\alpha-1}\left(\int_0^1 K(x,t)\,(\phi(t)^m)\,dt\right) ds\mathbf{B}$$

$$|Ty_N(x) - Ty_{N-1}(x)| = \frac{1}{\Gamma(\alpha)}\int_0^x (x-s)^{\alpha-1}\left(\int_0^1 K(x,t)\,(\phi(t)^m\ )\,|\mathbf{B}-\mathbf{A}|\,dt\right) ds$$

Since $x\epsilon\,[0,1]$ *and* $|\mathbf{B}-\mathbf{A}| \neq 0$, hence $\lim_{N\to 0}\Delta_N(x) \to 0$ Therefore the method of solution converges. □

## 6. Numerical Examples

In this section, we present numerical examples of linear and nonlinear to evaluate the effectiveness and accuracy of the method. Let $y_n(x)$ and $y(x)$ be the approximate and exact solutions, respectively. $\text{Error}_N = |y_n(x) - y(x)|$ **Example 1:** [29] Considering linear Fredholm integro-differential equation

$$y^{'}(x) + \frac{1}{3}y(x) - \int_0^1 xty(t)dt = f(x) \tag{17}$$

subject to initial condition

$$y(0) = 0$$

where

$$f\,(x) = 1$$

Exact solution: $y\,(x) = x$ The approximate solution of equation (17) at $N = 3$ gives

$$y_3 = 1.000000000000x + 3.552713678801\times10^{-15}x^2 - 3.552713678801\times10^{-15}x^3$$

**Table 1:** Exact, approximate and absolute error values for example 1

| $x$ | Exact | Our method$_{N=3}$ | error$_3$ | error [29]$_{N=3}$ |
|---|---|---|---|---|
| 0.2 | 0.200000000000 | 0.200000000000 | 0.00 | 8.32667e-17 |
| 0.4 | 0.400000000000 | 0.400000000000 | 0.00 | 2.22045e-16 |
| 0.6 | 0.600000000000 | 0.600000000000 | 0.00 | 1.11022e-16 |
| 0.8 | 0.800000000000 | 0.800000000000 | 0.00 | 1.11022e-16 |
| 1.0 | 1.000000000000 | 1.000000000000 | 0.00 | 1.11022e-16 |

**Example 2:** [29] Considering linear Fredholm integro-differential equation

$$y^{'}(x) - \int_0^1 xy(t)dt = f(x) \tag{18}$$

subject to initial condition

$$y(0) = 0$$

where

$$f(x) = xe^x + e^x - x$$

Exact solution: $y(x) = xe^x$ The approximate solution of equation (18) at $N = 3, 5$ *and* 7 gives

$$y_3 = \left(-5.610063457000 \times 10^{-12} + 1.014452579454x + 0.703408832104x^2 + 0.997286084761x^3\right)$$

$$y_5 = \begin{pmatrix} 1.484922630000 \times 10^{-14} + 1.000013757479x + 0.996267549909x^2 \\ +0.520765000052x^3 + 0.124449681936x^4 + 0.76776116165e - 1x^5 \end{pmatrix}$$

$$y_9 = \begin{pmatrix} -8.741312253000 \times 10^{-16} + 0.999999999995x + 0.999999925494x^2 \\ +0.500001162291x3 + 0.166660785675x^4 + 0.41699409485e - 1x^5 \\ +0.8277893066e - 2x^6 + 0.1466751099e - 2x^7 + 0.144958496e - 3x^8 + 0.44822693e - 4x^9 \end{pmatrix}$$

**Table 2:** Exact and approximate values for example 2

| $x$ | Exact | $N = 3$ | $N = 5$ | $N = 9$ |
|---|---|---|---|---|
| 0.2 | 0.244280551600 | 0.239005157800 | 0.244243261300 | 0.244280556400 |
| 0.4 | 0.596729879200 | 0.582152754100 | 0.596709370100 | 0.596729996700 |
| 0.6 | 1.093271280000 | 1.093248602000 | 1.222118351000 | 1.093272168000 |
| 0.8 | 1.780432742000 | 1.780386506000 | 1.425540168000 | 1.780436804000 |
| 1.0 | 2.718281828000 | 2.718272105000 | 1.718280607000 | 2.718295708000 |

**Table 3**: Absolute Error for example 2

| **x** | $\text{error}_3$ | $\text{error}_5$ | error9 | error [29] $_{N=10}$ |
|---|---|---|---|---|
| 0.2 | 5.275393800e-3 | 3.7290300e-5 | 4.800000000000e-9 | 2.80414e-6 |
| 0.4 | 1.4577125100e-2 | 2.0509100e-5 | 1.175000000000e-7 | 2.01829e-7 |
| 0.6 | 1.5958759000e-2 | 2.2678000e-5 | 8.880000000000e-7 | 2.80414e-6 |
| 0.8 | 8.078551000e-3 | 4.6236000e-5 | 4.062000e-7 | 2.01829e-6 |
| 1.0 | 3.134332000e-3 | 9.723000e-6 | 1.3880000e-7 | 5.44857e-6 |

**Example 3:** [16] Considering nonlinear Fredholm integro-differential equation

$$y'(x) - \int_0^1 x\,(y(t))^2\,dt = f(x) \tag{19}$$

subject to initial condition

$$y(0) = 0$$

where

$$f(x) = 1 - \frac{x}{3}$$

Exact solution: $y(x) = x$ The approximate solution of equation (19) at $N = 3$ gives

$$y_3 = 2.720046457648 \times 10^{-17} + 1.000000000000x$$

**Table 4:** Exact and approximate values for example 3

| $x$ | Exact | Our method$_{N=3}$ | $\text{error}_3$ | error [16]$_{N=3}$ |
|---|---|---|---|---|
| 0.2 | 0.200000000000 | 0.200000000000 | 0.00 | 5.86e-19 |
| 0.4 | 0.400000000000 | 0.400000000000 | 0.00 | 2.34e-18 |
| 0.6 | 0.600000000000 | 0.600000000000 | 0.00 | 5.27e-18 |
| 0.8 | 0.800000000000 | 0.800000000000 | 0.00 | 9.38e-18 |
| 1.0 | 1.000000000000 | 1.000000000000 | 0.00 | 1.47e-17 |

**Example 4** [27] Considering nonlinear Fredholm integro-differential equation

$$y^{'}(x) - \int_0^1 xte^{-(y(t))^2}dt = f(x) \tag{20}$$

subject to initial condition

$$y(0) = 0$$

where

$$f(x) = 1 - \frac{1}{2}x + \frac{x}{2e}$$

Exact solution: $y(x) = x$ The approximate solution of equation (20) at $N = 6$ gives

$$y_6 = \begin{pmatrix} -3.579410672000 \times 10^{-16} + 1.000000000x \\ +4.547473509000 \times 10^{-13}x^2 - 7.275957614000 \times 10^{-12}x^3 \\ +1.455191523000 \times 10^{-11}x^4 + 2.910383046000 \times 10^{-11}x^5 \end{pmatrix}$$

**Table 5:** Exact, approximate and absolute error values for example 4

| $x$ | Exact | Our method$_{N=6}$ | error$_6$ | error [27]$_{N=6}$ |
|---|---|---|---|---|
| 0.2 | 0.200000000000 | 0.200000000000 | 0.00 | 7.00e-7 |
| 0.4 | 0.400000000000 | 0.400000000000 | 0.00 | 3.00e-6 |
| 0.6 | 0.600000000000 | 0.600000000000 | 0.00 | 6.00e-6 |
| 0.8 | 0.800000000000 | 0.800000000000 | 0.00 | 1.00e-5 |
| 1.0 | 1.000000000000 | 1.000000000000 | 0.00 | 1.00e-5 |

## 7. **Discussion of Results**

In this section, we discuss the numerical results obtained from the solved examples using the derived numerical method. Based on the result obtained for example 1, as shown in Table 1, the approximate solution at N = 3 gives $y_3 = 1.000000000000x + 3.552713678801 \times 10^{-15}x^2 - 3.552713678801 \times 10^{-15}x^3$. The numerical result converged to an exact solution, and this confirmed that our method performed better than the method proposed by [29]. The results of numerical example 2 as shown in Tables 2 and 3 show that the approximate solution at N = 3 gives $y_3 = -5.610063457000 \times 10^{-12} + 1.014452579454x + 0.703408832104x^2 + 0.997286084761x^3$. Solving for N = 5 and 9, the numerical results converge to an exact solution as the value of N increases. This shows that the numerical method developed is consistent and gives a better result than the method proposed by [29]. The approximate solution obtained in example 3 at N = 3 gives $y_3 = 2.720046457648 \times 10^{-17} + 1.000000000000x$.The numerical result converged to an exact solution, and this confirmed that our method performed better than the method proposed by [16]. as shown in Table 4. In example 4, the approximate solution at $N = 6$ gives $y_6 = -3.579410672000 \times 10^{-16} + 1.000000000x + 4.547473509000 \times 10^{-13}x^2 - 7.275957614000 \times 10^{-12}x^3 + 1.455191523000 \times 10^{-11}x^4 + 2.910383046000 \times 10^{-11}x^5$. Table 5 shows the results obtained at $x = 0.2\ to\ 1.0$ for the value of N, the exact solution, and the absolute error. We observed that the numerical result also converged to the exact solution. This confirmed that the numerical method we developed is consistent and converges faster.

## 8. Conclusion

In this study, we investigated the collocation approach for the numerical solution of linear and nonlinear Fredholm integro-differential equations. This approach is simple to compute, dependable, and efficient. All of the calculations in this work are done using Maple 18. In this study, we investigated the collocation approach for the numerical solution of linear and nonlinear Fredholm integro-differential equations. This approach is simple to compute, dependable, and efficient. All of the calculations in this work are done using Maple 18.

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