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A STUDY OF N-FRACTIONAL CALCULUS FOR THE GENERALIZED HURWITZ-LERCH ZETA FUNCTION AND MITTAG-LEFFLER FUNCTION

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Abstract. In present paper, the investigation of generalized Hurwitz-Lerch Zeta function and generalized Mittag-Leffler function by applying N-fractional calculus has been discussed. Further, we establish the product of N-fractional calculus involving the generalized Hurwitz – Lerch Zeta function and Mittag-Leffler function. The main results provide useful extension and unification of a number of results for various types of functions. Further, several special cases are established at the end of the paper.

1. Introduction

We will be use some following definitions for our further investigation:

Definition 1.1. [11] *The generalized Hurwitz-Lerch Zeta function is introduced by Gupta and defined as*

$$\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b) = \frac{1}{\Gamma(s)} \sum_{m=0}^{\infty} \frac{(-b)^m \Gamma(s-\alpha m)}{m!} \times \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{r} (\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{s} (\mu_k)_{\sigma_k n}} \frac{z^n}{(a+n\beta)^{s-\alpha m} n!} \tag{1}$$

where $\beta, s, z, \lambda_j \in \mathbb{C}(j = 1,\dots,p), \mu_k \in \mathbb{C}/Z_0^-(k = 1,\dots,q), \rho_j, \sigma_k \in R^+(j = 1,\dots,p; k = 1,\dots,q), p, q > 0, \mathrm{Re}(b) \geq 0.$

If we substitute $b = 0$ *and* $\alpha = \beta = 1$ *in* (1)*, we get multi-parameter Hurwitz-Lerch Zeta function is given by Srivastava* [26].

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On substituting $q = b = 0, \alpha = \beta = 1, \rho = \rho_1 = \lambda_1 = 1$ *and* $q = \sigma_1 = \mu_1 = 1$ *in* (1)*, the function reduces to the extended Hurwitz-Lerch Zeta function given by Lin and Srivastava* [16]*. Further, it reduces to the generalized Hurwitz-Lerch Zeta function given by Podlubny* [20] *when* $\rho = 1, \sigma = 1$ *and* $\lambda = 1$*. which further reduces to the Hurwitz-Lerch zeta function given by Garg et. al.* [10] *if* $\mu = 1$*.*

Definition 1.2. [15] *The generalized Mittag-Leffler function is introduced by Salim* [23] *(see also) and defined as*

$$E_{\theta,\vartheta}^{\gamma,\delta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(n\theta+\vartheta)} \frac{z^n}{(\delta)_n} \tag{2}$$

where $z, \theta, \vartheta, \delta \in \mathbb{C}, \mathcal{R}e(\theta) > 0, \mathcal{R}e(\vartheta) > 0, \mathcal{R}e(\gamma) > 0, \mathcal{R}e(\delta) > 0$*.*

On setting $\delta = 1$ *in (2), we get another form of generalized Mittag-Leffler function, which is given by Prabhakar* [21]*. It reduces to Mittag-Leffler function introduced by Wiman* [32] *if* $\gamma = 1$*. Further, it reduces to the generalized Mittag-Leffler defined by Mittag-Leffler* [17] *when* $\vartheta = 1$*.*

Recent work on Mittag-Leffler function and fractional calculus have been studied in references to its significance and applicability in various fields [1]-[9]*,* [12]-[15]*,* [18]*,* [22]*,* [24]*,* [27]-[28]*.*

Definition 1.3. [19] *The* N*-fractional calculus is introduced by Nishimoto and defined in the following manner:*

Let $D = \{D_-, D_+\}, C = \{C_-, C_+\}, C_-$ *be a curve along the cut joining two points* z *and* $-\infty + i\,\mathrm{Im}(z), C_+$ *be a curve along the cut joining two points* z *and* $\infty + i\,\mathrm{Im}(z). D_-$ *and* D_+ *are domains surrounded by* C_- *and* C_+ *respectively.* D_+ *contains two points over the curve* C_+*.*

Further, let $f = f(z)$ *be a regular function in* $D(z \in D)$*. Then*

$$f_n = (f)_n = \frac{\Gamma(n+1)}{2\pi i} \int_C \frac{f(\zeta)}{(\zeta - z)^{n+1}} d\zeta, \left(n \neq Z^- := \{-1, -2, -3 \ldots ..\}\right) \tag{3}$$

$$(f)_{-r'} = \lim_{n \to -r'} (f)_n, \left(r' \in Z^+ := \{1, 2, 3 \ldots\}\right) \tag{4}$$

$$(z^s)_n = e^{-i\pi n} \frac{\Gamma(n-s)}{\Gamma(-s)} z^{s-n} \left[0 \neq \left|\frac{\Gamma(n-s)}{\Gamma(-s)}\right| < \infty\right] \tag{5}$$

where $-\pi \leq \arg(\zeta - z) \leq \pi$ *for* $C_-, 0 \leq \arg(\zeta - z) \leq 2\pi$ *for* $C_+, \zeta \neq z, z \in C$ *and* $n \in C$*, then for* $\mathrm{Re}(n) > 0, (f)_n$ *is derivative of arbitrary order* n *and for* $\mathrm{Re}(n) < 0$*, integral of arbitrary order* $-n$ *with respect to* z *of the function* $f(z)$*.*

$$(1+z)^{-\lambda} = \sum_{k=0}^{\infty} \frac{(\lambda)_k (-z)^k}{k!} |z| < 1. \tag{6}$$

2. Main Results

We assess four theorems for N-fractional calculus including the generalized Hurwitz - Lerch Zeta function and generalized Mittag-Leffler function in this part.

Theorem 2.1. *Let the following conditions are satisfied:*

(i) $0 \neq \left|\frac{\Gamma(v-p-n)}{(-p-n)}\right| < \infty$

(ii) $\left|\frac{n\beta}{a}\right| < 1.$

(iii) The conditions defined with (1) are fulfilled. Then, the outcomes holds as under

$$\left(z^p\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,u,a,b)\right)_v$$
$$= \frac{(-1)^v}{a^s\Gamma(s)}\sum_{m=0}^{\infty}\sum_{n,k=0}^{\infty}\frac{\Gamma(s-\alpha m)\Gamma(v-n-p)(-b)^m(a+n\beta)^{\alpha m}}{\Gamma(-n-p)m!n!k!}\frac{\prod_{j=1}^{r}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{s}(\mu_k)_{\sigma_k n}}$$
$$\times (s)_k\left(-\frac{n\beta}{a}\right)^k z^{n+p-v} \tag{7}$$

Proof. For convenience, the left hand side of (7) assumes $\mathcal{L}_1$ and to prove this, first we express the generalized Hurwitz lerch zeta function in terms of series form by using (1), we have

$$\mathcal{L}_1 = \left(\frac{a^{-s}}{\Gamma(s)}\sum_{m=0}^{\infty}\frac{(-b)^m\Gamma(s-\alpha m)}{m!}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{r}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{s}(\mu_k)_{\sigma_k n}}\frac{(a+n\beta)^{\alpha m}\left(1+\frac{n\beta}{a}\right)^{-s}z^{n+p}}{n!}\right)_v.$$

Using the result (6) in above result, we get

$$\mathcal{L}_1 = \left(\frac{a^{-s}}{\Gamma(s)}\sum_{m=0}^{\infty}\frac{(-b)^m\Gamma(s-\alpha m)}{m!}\sum_{n,k=0}^{\infty}\frac{\prod_{j=1}^{r}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{s}(\mu_k)_{\sigma_{kn}}}\frac{(a+n\beta)^{\alpha m}(s)_k\left(-\frac{n\beta}{a}\right)^k z^{n+p}}{n!k!}\right)_v.$$
$$= \frac{a^{-s}}{\Gamma(s)}\sum_{m=0}^{\infty}\sum_{n,k=0}^{\infty}\frac{(-b)^m(a+n\beta)^{\alpha m}\Gamma(s-\alpha m)}{m!}\frac{\prod_{j=1}^{r}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{s}(\mu_k)_{\sigma_k n}}\frac{(u)_k\left(-\frac{n\beta}{a}\right)^k}{n!k!}\left(z^{n+p}\right)_v$$

Now, applying the result (5), we have

$$\mathcal{L}_1 = (-1)^v$$
$$\frac{a^{-s}}{\Gamma(s)}\sum_{m=0}^{\infty}\sum_{n,k=0}^{\infty}\frac{(-b)^m(a+n\beta)^{\alpha m}\Gamma(s-\alpha m)}{m!n!k!}\frac{\prod_{j=1}^{r}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{s}(\mu_k)_{\sigma_k n}}\frac{(s)_k\left(-\frac{n\beta}{a}\right)^k}{\frac{\Gamma(v-n-p)}{\Gamma(-n-p)}z^{n+p-v}}.$$

We obtain the necessary outcome (7) after a little rearrangement. □

Theorem 2.2. *Let the following conditions are fulfilled:*

(i) $0 \neq \left|\frac{\Gamma(v+v'-n-\rho)}{\Gamma(v-n-\rho)}\right| < \infty, 0 \neq \left|\frac{\Gamma(v+v'-n-\rho)}{\Gamma(v'-n-\rho)}\right| < \infty, 0 \neq \left|\frac{\Gamma(v+v'-n-\rho)}{\Gamma(-n-\rho)}\right| < \infty$

(ii) $\left|\frac{n\beta}{a}\right| < 1.$

(iii) The condition defined above (2.2) are satisfied the result hold as under

$$\left(\left(z^p\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b)\right)_v\right)'_v = \left(\left(z^p\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b)\right)_{v'}\right)_v$$
$$= \left(z^p\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b)\right)_{v+v'} \tag{8}$$

Proof. In order to prove (8), from (7), we have

$$\left(\left(z^{p}\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b)\right)_{v}\right)v'$$
$$=\frac{(-1)^{v}}{a^{s}\Gamma(s)}\sum_{m=0}^{\infty}\frac{(-b)^{m}\Gamma(s-\alpha m)}{m!}$$
$$\times\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{q}(\mu_k)_{\sigma_{kn}}}\frac{(a+n\beta)^{\alpha m}(s)_k}{n!k!}\left(\frac{-n\beta}{a}\right)^{k}\frac{\Gamma(v-n-\rho)}{\Gamma(-n-\rho)}\left(z^{n+\rho-v}\right)_{v'}. \tag{9}$$

Using (6) in (9), we get

$$=\frac{(-1)^{v+v'}}{a^{s}\Gamma(s)}\sum_{m=0}^{\infty}\frac{(-b)^{m}\Gamma(s-\alpha m)}{m!}$$
$$\times\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_{jn})\,\rho_{jn}}{\prod_{k=1}^{q}(\mu_k)_{\sigma_k n}}\frac{(a+n\beta)^{\alpha m}(s)_k}{n!k!}\left(\frac{-n\beta}{a}\right)^{k}\frac{\Gamma\left(v+v'-n-\rho\right)}{\Gamma(-n-\rho)}z^{n+\rho-v-v'}.$$

Similarly, we obtain

$$\left(\left(z^{p}\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b)\right)_{v'}\right)_{v}=\frac{(-1)^{v+v'}}{a^{s}\Gamma(s)}\sum_{m=0}^{\infty}\frac{(-b)^{m}\Gamma(s-\alpha m)}{m!}$$
$$\times\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{q}(\mu_k)_{\sigma_k n}}\frac{(a+n\beta)^{\alpha m}(s)_k}{n!k!}\left(\frac{-n\beta}{a}\right)^{k}\frac{\Gamma\left(v+v'-n-\rho\right)}{\Gamma(-n-\rho)}z^{n+\rho-v-v'}$$

Replacing v by $v+v$ in (7), we get

$$\left(z^{p}\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b)\right)_{\nu+v'}$$
$$=\frac{(-1)^{v+v'}}{a^{s}\Gamma(s)}\sum_{m=0}^{\infty}\frac{(-b)^{m}\Gamma(s-\alpha m)}{m!}$$
$$\times\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_j)\,\rho_{jn}}{\prod_{k=1}^{q}(\mu_k)_{\sigma_k n}}\frac{(a+n\beta)^{\alpha m}(s)_k}{n!k!}\left(\frac{-n\beta}{a}\right)^{k}\frac{\Gamma\left(v+v'-n-\rho\right)}{\Gamma(-n-\rho)}z^{n+\rho-v-v'}.$$

□

Theorem 2.3. *Let the following conditions are fulfilled:*

(i) $0\neq\left|\frac{\Gamma(\nu-n-\rho-\varepsilon n)}{(-n-\rho-\epsilon n)}\right|<\infty$

(ii) $\left|\frac{n\beta}{a}\right|<1.$

(iii) The condition defined above (2.3) are satisfied the result hold as under

$$\left(z^{p}\Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b)E^{\gamma,\delta}_{\theta,\vartheta}\left(\eta z^{\varepsilon}\right)\right)_{v}=\frac{(-1)^{v}}{a^{s}}\frac{1}{\Gamma(s)}\sum_{m=0}^{\infty}\frac{(-b)^{m}\Gamma(s-\alpha m)}{m!}$$
$$\times\sum_{n,k,s=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{q}(\mu_k)_{\sigma_k n}}\frac{(a+n\beta)^{\alpha m}(s)_k(\gamma)_n\left(\eta z^{\varepsilon}\right)^{n}}{n!k!\Gamma(n\theta+\vartheta)(\delta)_n}\left(\frac{-n\beta}{a}\right)^{k}$$
$$\times\frac{\Gamma(v-n-\rho-\varepsilon n)}{\Gamma(-n-\rho-\varepsilon n)}z^{n+\rho-v}.$$

Proof. The theorem can be easily be derived from Theorem 2.1 So, the details are omitted. □

Theorem 2.4. *Let the following conditions are fulfilled:*

(i) $0 \neq \left|\frac{\Gamma\left(v+v'-n-\rho-\varepsilon n\right)}{\Gamma(v-n-\rho-\varepsilon n)}\right| < \infty, 0 \neq \left|\frac{\Gamma\left(v+v'-n-\rho-\varepsilon n\right)}{\Gamma(v'-n-\rho-\varepsilon n)}\right| < \infty, 0 \neq \left|\frac{\Gamma\left(v+v'-n-\rho-\varepsilon n\right)}{\Gamma(-n-\rho-\varepsilon n)}\right| < \infty$

(ii) $\left|\frac{n\beta}{a}\right| < 1.$

(iii) The condition defined above (2.4) are satisfied the the result hold as under

$$\left(\left(z^p \Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b) E^{\gamma,\delta}_{\theta,\vartheta}\left(\eta z^{\varepsilon}\right)\right)_v\right)_{v'} = \left(\left(z^p \Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b) E^{\gamma,\delta}_{\theta,\vartheta}\left(\eta z^{\varepsilon}\right)\right)_{v'}\right)_v$$
$$= \left(z^p \Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b) E^{\gamma,\delta}_{\theta,\vartheta}\left(\eta z^{\varepsilon}\right)\right)_{v+v'}$$

Proof. The theorem can be easily be derived from Theorem 2.1 So, the details are omitted. ☐

3. Special Cases

Corollary 3.0. [26] *In Theorem 2.1, If we put $b = 0$ and $\alpha = \beta = 1$, we get the following result contains multi-parameter Hurwitz-Lerch Zeta function as under:*

$$\left(z^p \Phi^{(\rho_p,\sigma_q)}_{(\lambda_q,\mu_q)}(z,s,a,b)\right)_v = \frac{(-1)^v}{a^s} \sum_{n,k=0}^{\infty} \frac{\prod_{j=1}^{p}\left(\lambda_j\right)_{\rho_{jn}}}{\prod_{k=1}^{q}\left(\mu_k\right)_{\sigma_k n}} \frac{1}{n!k!} \left(\frac{-n\beta}{a}\right)^k \frac{\Gamma(v-n-\rho)}{\Gamma(-n-\rho)} z^{n+\rho-v}. \tag{10}$$

Corollary 3.0. [20] *If we substitute $q = b = 0$ and $\alpha = \beta = 1, p = \rho_1 = \lambda_1 = 1, q = \sigma_1 = \mu_1 = 1, \rho = \sigma = 1$ and $\lambda = 1$ in* Theorem 2.1*, then we get the following result contains generalized Hurwitz-Lerch Zeta function as below:*

$$\left(z^p \Phi^*_{\mu}(z,s,a)\right)_v = \frac{(-1)^v}{a^s} \sum_{n,k=0}^{\infty} \frac{(\mu)_n}{n!k!} \left(\frac{-n}{a}\right)^k \frac{\Gamma(v-n-\rho)}{\Gamma(-n-\rho)} z^{n+\rho-v}$$

Corollary 3.0. [10] *If we substitute $b = 0$ and $\alpha = \beta = 1, p = \rho_1 = \lambda_1 = 1, q = \sigma_1 = \mu_1 = 1$, $\rho = \sigma = \lambda = 1$ and $\mu = 1$ in* Theorem 2.1*, then we get the following result contains HurwitzLerch Zeta function as follow:*

$$\left(z^p(z,s,a)\right)_v = \frac{(-1)^v}{a^s} \sum_{n,k=0}^{\infty} \frac{1}{k!} \left(\frac{-n}{a}\right)^k \frac{\Gamma(v-n-\rho)}{\Gamma(-n-\rho)} z^{n+\rho-v}.$$

Corollary 3.0. [21] *If we take $\delta = 1$ in Theorem 2.3, then we get the following result contains generalized Mittag-Leffler function as under:*

$$\left(z^p \Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b) E^{\gamma}_{\theta,\vartheta}\left(\eta z^{\varepsilon}\right)\right)_v = \frac{(-1)^v}{a^s} \frac{1}{\Gamma(s)} \sum_{m=0}^{\infty} \frac{(-b)^m \Gamma(s-\alpha m)}{m!}$$
$$\times \sum_{n,k,s=0}^{\infty} \frac{\prod_{j=1}^{p}\left(\lambda_j\right)_{\rho_{jn}}}{\prod_{k=1}^{q}\left(\mu_k\right)_{\sigma_k n}} \frac{(a+n\beta)^{\alpha m}(\gamma)_n (s)_k \left(\eta z^{\varepsilon}\right)^n}{n!^2 k! \Gamma(n\theta+\vartheta)} \left(\frac{-n\beta}{a}\right)^k \frac{\Gamma(v-n-\rho-\varepsilon n)}{\Gamma(-n-\rho-\varepsilon n)} z^{n+\rho-v}$$

Corollary 3.0. [32] *In Theorem 2.3, if we take $\delta = \gamma = 1$ then we get the following result contains generalized Mittag- Leffler function as follow:*

$$\left(z^p \Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b) E_{\theta,\vartheta}\left(\eta z^{\varepsilon}\right)\right)_v = \frac{(-1)^v}{a^s} \frac{1}{\Gamma(s)} \sum_{m=0}^{\infty} \frac{(-b)^m \Gamma(s-\alpha m)}{m!}$$
$$\times \sum_{n,k,s=0}^{\infty} \frac{\prod_{j=1}^{p}\left(\lambda_j\right)_{\rho_{jn}}}{\prod_{k=1}^{q}\left(\mu_k\right)_{\sigma_k n}} \frac{(a+n\beta)^{\alpha m} (s)_k \left(\eta z^{\varepsilon}\right)^n}{n!k! \Gamma(n\theta+\vartheta)} \left(\frac{-n\beta}{a}\right)^k \frac{\Gamma(v-n-\rho-\varepsilon n)}{\Gamma(-n-\rho-\varepsilon n)} z^{n+\rho-v}$$

Corollary 3.0. [17] *In Theorem 2.3, if we put $\delta = \gamma = \vartheta = 1$, then we get the following result contains generalized Mittag- Leffler function as follow:*

$$\left(z^p \Phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b) E_\theta\left(\eta z^\varepsilon\right)\right)_v = \frac{(-1)^v}{a^s}\frac{1}{\Gamma(s)}\sum_{m=0}^{\infty}\frac{(-b)^m \Gamma(s-\alpha m)}{m!}$$

$$\times \sum_{n,k,s=0}^{\infty} \frac{\prod_{j=1}^{p}(\lambda_j)_{\rho_{jn}}}{\prod_{k=1}^{q}(\mu_k)_{\sigma_k n}} \frac{(a+n\beta)^{\alpha m}(s)_k \left(\eta z^\varepsilon\right)^n}{n!k!\Gamma(n\theta+1)} \left(\frac{-n\beta}{a}\right)^k \frac{\Gamma(v-n-\rho-\varepsilon n)}{\Gamma(-n-\rho-\varepsilon n)} z^{n+\rho-v}.$$

A number of several other results can also be obtained by putting some specific value of parameters.

4. CONCLUSION

We have established four theorems following to N-fractional calculus of product of generalized Mittag-Leffler function and generalized Hurwitz -Lerch Zeta function. The major outcomes of the work is to explore useful extension and unification of number of results for various types of functions. Also, we mentioned some corollaries as special cases of the main results.

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