Journal of Fractional Calculus and Applications
Vol. 12(3). No.10, pp. 1-13
1st. Inter. E-Conf. in Math. Sciences and Fractional Calculus(ICMSFC Feb 2021).
ISSN: 2090-5858.
http://math-frac.org/Journals/JFCA/

WAVELET BASED LIFTING SCHEMES FOR THE NUMERICAL SOLUTION OF NEWELL-WHITEHEAD-SEGEL EQUATIONS

L. M. ANGADI

Abstract. The Newell-Whitehead-Segel equations have wide applicability in mechanical and chemical engineering, ecology, biology and bio-engineering. In this paper, we presented Lifting schemes using different wavelet filter coefficients for the numerical solution of linear and non-linear Newell-Whitehead-Segel equations. The numerical results obtained by these schemes are compared with the exact solution to demonstrate the accuracy and also faster convergence in lesser computational time as compared with existing scheme. Some test problems are taken to demonstrate the applicability and accuracy of the scheme.

1. Introduction

Nonlinear phenomena play a vital role in applied mathematics and physics, we know that for the most part of engineering problems is non-linear, and it is difficult to solve them analytically. The importance of obtaining the exact or approximate solutions of nonlinear partial differential equations (PDEs) in physics and mathematics is still a significant problem that desires new methods to discover exact or approximate solutions [1].

Newell-Whitehead-Segel equation is one of the most important of amplitude equations, which describes the appearance of the stripe pattern in two dimensional systems. Moreover, this equation was applied to a number of problems in a variety system, e.g., Rayleigh-Bernard convection, Faraday instability, nonlinear optics, chemical reactions and biological systems. The approximate solutions of the Newell-Whitehead-Segel equation were presented by Adomain decomposition [2], differential transformation [3] etc.

Wavelet analysis assumed significance due to successful applications in signal and image processing during the 1980s. The smooth orthonormal bases obtained by the translation and dilation of a single function in a hierarchical fashion proved

---

2010 *Mathematics Subject Classification.* 65T60, 97N40, 34A08.
Key words and phrases. Newell-Whitehead-Segel equations, Lifting schemes; Orthogonal and Biorthogonal wavelets.
Submitted Nov. 13, 2020.

very useful to develop compression algorithms for signals and images up to a chosen threshold of relevant amplitudes. Some of the major contributors to this theory are: Multiresolution signal processing used in computer vision; sub band coding, developed for speech and image compression; and wavelet series expansion, developed in applied mathematics. Wavelets permit the perfect representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithm [4].

In recent times, some of the works on wavelet based methods are the discrete wavelet transforms (DWT) and the full approximation scheme (FAS) were introduced recently in ([5] - [6]]). The wavelet based full approximation scheme (WFAS) has exposed to be a very efficient and favorable method for numerous problems related to computational science and engineering fields [7]. These methods can be either used as an iterative solver or as a preconditioning technique, offering in many cases a better performance than some of the most innovative and existing FAS algorithms. Due to the efficiency and potentiality of WFAS, researches further have been carried out for its enrichment. In order to realize this task, work build that is orthogonal/biorthogonal discrete wavelet transform using lifting scheme [8]. Wavelet based lifting technique is introduced by Sweldens [9], which permits some improvements on the properties of existing wavelet transforms. Wavelet based numerical solution of elasto-hydrodynamic lubrication problems via lifting scheme was introduced by Shiralashetti et al. [10]. The technique has some numerical benefits as a reduced number of operations which are fundamental in the context of the iterative solvers. Evidently all attempts to simplify the wavelet solutions for PDE are welcome. In PDE, matrices arising from system are dense with non-smooth diagonal and smooth away from the diagonal. This smoothness of the matrix transforms into smallness using wavelet transform and it leads to design the effective wavelets based lifting scheme.

Lifting scheme is a new approach to construct the so-called second-generation wavelets that are not necessarily translations and dilations of one function. The latter we refer to as a first-generation wavelets or classical methods. The lifting scheme has some additional advantages in comparison with the classical wavelets. This transform works for signals of an arbitrary size with correct treatment of boundaries. Another feature of the lifting scheme is that all constructions are derived in the spatial domain. This is in contrast to the traditional approach, which relies heavily on the frequency domain.

The lifting scheme starts with a set of well-known filters, thereafter lifting steps are used an attempt to improve (lift) the properties of a corresponding wavelet decomposition. This procedure has some mathematical benefits as a reduced number of operations which are essential in the context of the iterative solvers. In addition to this, the present paper illustrates that the application of the lifting scheme for the numerical solution of linear and non-linear Newell-Whitehead-Segel equations.

The present paper is organized as follows: Preliminaries of wavelet filter coefficients and lifting scheme are given in section 2. In section 3, the method of solution is explained. Numerical results of the problems are presented in section 4 and finally, conclusion of the proposed work is given in section 5.

2. Preliminaries of wavelet filter coefficients and Lifting scheme

The lifting scheme starts with a set of well-known filters; thereafter lifting steps are used in attempt to improve the properties of corresponding wavelet decomposition. Now, we have discussed about different wavelet filters as follows:

2.1. **Haar wavelet filter coefficients.**
We know that low pass filter coefficients $[a_0,\ a_1]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ and high pass filter coefficients $[b_0,\ b_1]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ play an important role in decomposition.

2.2. **Daubechies wavelet filter coefficients.**

Daubechies introduced scaling functions having the shortest possible support. The scaling function ϕ_N has support$[0,\ N-1]$, while the corresponding wavelet ψ_N has support in the interval$\left[1\ -\ \frac{N}{2}\ ,\ \ \frac{N}{2}\right]$.
We have low pass filter coefficients $[a_0,\ a_1,\ \ a_2,\ a_3]^T = \left[\frac{1+\sqrt{3}}{4\sqrt{2}},\ \ \frac{3+\sqrt{3}}{4\sqrt{2}},\ \ \frac{3-\sqrt{3}}{4\sqrt{2}},\ \ \frac{1-\sqrt{3}}{4\sqrt{2}}\right]^T$ and high pass filter coefficients $[b_0,\ \ b_1,\ \ b_2,\ b_3]^T = \left[\frac{1-\sqrt{3}}{4\sqrt{2}},\ \ -\frac{3-\sqrt{3}}{4\sqrt{2}},\ \ \frac{3+\sqrt{3}}{4\sqrt{2}},\ \ -\frac{1+\sqrt{3}}{4\sqrt{2}}\right]^T$

2.3. **Biorthogonal (CDF (2,2)) wavelets.**

Let's consider the (5, 3) biorthogonal spline wavelet filter pair, the low pass filter pair are $(\tilde{a}_{-1}, \tilde{a}_0, \tilde{a}_1) = \left(\frac{1}{2\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$ and $(a_{-2}, a_{-1}, a_0, a_1, a_2) = \left(\frac{-1}{4\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{-1}{4\sqrt{2}}\right)$.
But, we have $b_k = (-1)^k\tilde{a}_{1-k}$ and $\tilde{b}_k = (-1)^k a_{1-k}$, the high pass filter pair are $b_0 = \frac{1}{2\sqrt{2}},\ b_1 = \frac{-1}{\sqrt{2}},\ b_2 = \frac{1}{2\sqrt{2}}$ & $\tilde{b}_{-1} = \frac{1}{4\sqrt{2}},\ \tilde{b}_0 = \frac{1}{2\sqrt{2}},\ \tilde{b}_1 = \frac{-3}{2\sqrt{2}},\ \tilde{b}_2 = \frac{1}{2\sqrt{2}},\ \tilde{b}_3 = \frac{1}{4\sqrt{2}}$

2.4. **Foundations of lifting scheme.**

Consider to numbers $a,\ b$ as two neighboring samples of a sequence and then these have some correlation which we would like to take advantage. The simple linear transform which replaces a and b by average $\boldsymbol{s}$ and difference d i.e.

$$s = \frac{a\ +\ b}{2}\quad \&\quad d = \frac{a\ -\ b}{2}$$

The idea is that if a and b are highly correlated, the expected absolute value of their difference d will be small and can be represented with fever bits. In case that $a = b$, the difference is simply zero. We have not lost any information because we can always recover a and b from the gives s and d as:

$$a = s\ -\ \frac{d}{2}\quad \&\quad b =\ s\ +\ \frac{d}{2}$$

Finally, a wavelet transforms built through lifting consists of three steps: split. Predict and update as given in the Figure 1 [10].
Split: Splitting the signal into two disjoint sets of samples.
Predict: If the signal contains some structure, then we can expect correlation between a sample and its nearest neighbors. i. e. $d_{j-1} = \mathrm{odd}_{j-1} - \mathrm{P}(\mathrm{even}_{j-1})$

Update: Given an even entry, we have predicted that the next odd entry has the same value, and stored the difference. We then update our even entry to reflect our knowledge of the signal. i.e. $s_{j-1} = \text{even}_{j-1} + \text{U}(d_{j-1})$
The detailed algorithm using different wavelets is given in the next section. The general lifting stages for decomposition and reconstruction of a signal are given in Figure 2.

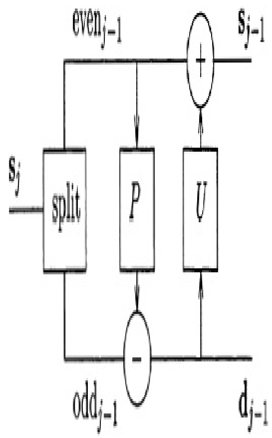

Fig. 1. Steps in lifting scheme

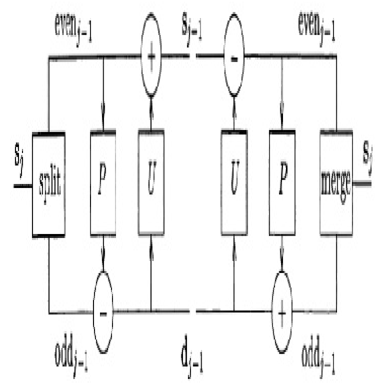

Fig. 2. Lifting wavelet algorithm

The detailed algorithm using different wavelets is given in the next section.

3. Method of Solution

Consider the general Newell-Whitehead-Segel equation

$$u_t = u_{xx} + au - bu^n, \; 0 \leq x \leq 1 \; \& \; t > 0 \qquad (1)$$

Where a , b are constants and n is positive integer.
After discretizing the equation (2) through the finite difference method (FDM), we get system of algebraic equations. Through this system we can write the system as

$$A\,u = b \qquad (2)$$

where A is $N \times N$ coefficient matrix, b is $N \times N$ matrix and u is $N \times N$ matrix to be determined.
where $N = 2^J$, N is the number of grid points and J is the level of resolution. Solve Eq. (2) through the iterative method, we get approximate solution. Approximate solution containing some error, therefore required solution equals to sum of approximate solution and error. There are many methods to minimize such error to get the accurate solution. Some of them are HWLS, DWLS, BWLS etc. Now we are using the advanced technique based on different wavelets called as lifting scheme. Recently, lifting schemes are useful in the signal analysis and image processing in the area of science and engineering. But currently it extends to approximations in the numerical analysis [6]. Here, we are discussing the algorithm of the lifting schemes as follows:

3.1. Haar wavelet Lifting scheme (HWLS).

Daubechies and Sweldens have shown that every wavelet filter can be decomposed into lifting steps. More details of the advantages as well as other important structural advantages of the lifting technique can be available in [8]. The representation of Haar wavelet via lifting form presented as;

Decomposition:
Consider approximate solution $S = \tilde{P}_j$ like as signal and then apply the HWLS decomposition (finer to coarser) procedure as,

$$\left.\begin{array}{l} d^{(1)} = S_{2j} - S_{2j-1}, \quad s^{(1)} = S_{2j-1} + \frac{1}{2}d^{(1)}, \quad S_1 = \sqrt{2}\, s^{(1)} \\ \text{and} \quad D = \frac{1}{\sqrt{2}} d^{(1)} \end{array}\right\} \tag{3}$$

In this stage finally, we get new approximation as,

$$S = [S_1 \ \ D] \tag{4}$$

Reconstruction:
Consider Eq. (2) and then apply the HWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{array}{l} d^{(1)} = \sqrt{2}\, D, \quad s^{(1)} = \frac{1}{\sqrt{2}} S_1, \quad S_{2j-1} = s^{(1)} - \frac{1}{2}d^{(1)} \\ \text{and} \quad S_{2j} = d^{(1)} + S_{2j-1} \end{array}\right\} \tag{5}$$

which is the required solution of the given equation.

3.2. Daubechies wavelet Lifting scheme (DWLS).

As discussed in the previous section 3.1, we follow the same procedure but we used different wavelet i.e., Daubechies 4^{th} order wavelet coefficient. The DWLS procedure is as follows;

Decomposition:

$$\left.\begin{array}{l} s^{(1)} = S_{2j-1} + \sqrt{3}\, S_{2j}, \quad d^{(1)} = S_{2j} - \frac{\sqrt{3}}{4} s^{(1)} - \left(\frac{\sqrt{3}-2}{4}\right) s_1^{(j-1)}, \\ s^{(2)} = s^{(1)} - d_1^{(j+1)}, \quad S_1 = \frac{\sqrt{3}-1}{\sqrt{2}} s^{(2)} \ \text{and} \\ D = \frac{\sqrt{3}+1}{\sqrt{2}} d^{(1)} \end{array}\right\} \tag{6}$$

Here, we get new approximation as,

$$S = [S_1 \ \ D] \tag{7}$$

Reconstruction:
Consider Eq. (5), then apply the DWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{array}{l} d^{(1)} = \frac{\sqrt{2}}{\sqrt{3}+1} D, \\ s^{(2)} = \frac{\sqrt{2}}{\sqrt{3}-1} S_1, \\ s_1^{(j)} = s^{(2)} + d_1^{(j+1)}, \\ S_{2j} = d^{(1)} + \frac{\sqrt{3}}{4} s_1^{(j)} + \frac{\sqrt{3}-2}{4} s_1^{(j-1)} \ \text{and} \\ S_{2j-1} = s^{(1)} - \sqrt{3}\, S_{2j} \end{array}\right\} \tag{8}$$

which is the required solution of the given equation.

3.3. Biorthogonal wavelet Lifting scheme (BWLS).

As discussed in the previous sections 3.1 and 3.2, we follow the same procedure here we used another wavelet i.e., biorthogonal wavelet (CDF (2,2)). The BWLS procedure is as follows;

Decomposition:

$$\left.\begin{aligned} d^{(1)} &= S_{2j} - \tfrac{1}{2}\left[S_{2j-1} + S_{2j+2}\right], \\ s^{(1)} &= S_{2j-1} + \tfrac{1}{4}\left[d^{(1)}_{j-1} + d^{(1)}\right], \\ D &= \tfrac{1}{\sqrt{2}} d^{(1)}, \\ S_1 &= \sqrt{2}\, s^{(1)} \end{aligned}\right\} \tag{9}$$

In this stage finally, we get new signal as,

$$S = [S_1 \;\; D] \tag{10}$$

Reconstruction:

Consider Eqn. (10), then apply the DWLS reconstruction (coarser to finer) procedure as

$$\left.\begin{aligned} s^{(1)} &= \tfrac{1}{\sqrt{2}} S_1, \\ d^{(1)} &= \sqrt{2}\, D, \\ S_{2j-1} &= s^{(1)} - \tfrac{1}{4}\left[d^{(1)}_{j-1} + d^{(1)}\right] \\ S_{2j} &= d^{(1)} + \tfrac{1}{2}\left[S_{2j-1} + S_{2j+2})\right], \end{aligned}\right\} \tag{11}$$

which is the required solution of the given equation.
The coefficients $s_1^{(j)}$ and $d_1^{(j)}$ are the average and detailed coefficients respectively of the approximate solution u_a . The new approaches are tested through some of the numerical problems and the results are shown in next section.

4. Numerical illustration

In this section, we applied Lifting scheme for the numerical solution of Newell-Whitehead-Segel equations and also show the validity and applicability of HWLS, DWLS and BWLS.
The error is computed by $E_{max} = \max \; |u_e(x, t) - u_a(x, t)|$, where $u_e(x, t)$ and $u_a(x, t)$ are exact and approximate solution respectively.
Problem 4.1: Consider the linear Newell-Whitehead-Segel equation,
$(In \;\; Eq.(1), \; a = 0, \; b = 3, \; n = 1)$

$$\text{i.e.} \quad u_t = u_{xx} - 3u, \quad 0 \le x \le 1 \;\; \& \;\; t > 0 \tag{12}$$

subject to the I.C.:

$$u(x, 0) = e^{2x} \tag{13}$$

and B.C.s:

$$\left.\begin{aligned} u(0, t) &= e^{t} \\ u(1, t) &= e^{2+t} \end{aligned}\right\} \tag{14}$$

Which has the exact solution $u(x, t) = e^{2x+t}$ [11].
By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solutions are presented in table 1, figure 3 and figure 4. The maximum absolute errors with CPU time of the methods are presented in table 2.

Table 1 Comparison of numerical solutions with exact solution of the problem 4.1.

x	t	Numerical solution				Exact solution
		FDM	HWLS	DWLS	BWLS	
0.2	0.2	1.83988	1.83988	1.83988	1.83988	1.82212
0.4		2.74874	2.74874	2.74874	2.74874	2.71828
0.6		4.09211	4.09211	4.09211	4.09211	4.05519
0.8		6.08092	6.08092	6.08092	6.08092	6.04965
0.2	0.4	2.25286	2.25286	2.25286	2.25286	2.22554
0.4		3.36685	3.36685	3.36685	3.36685	3.32012
0.6		5.00848	5.00848	5.00848	5.00848	4.95303
0.8		7.43441	7.43441	7.43441	7.43441	7.38905
0.2	0.6	2.75335	2.75335	2.75335	2.75335	2.71828
0.4		4.11509	4.11509	4.11509	4.11509	4.05519
0.6		6.12029	6.12029	6.12029	6.12029	6.04965
0.8		9.08229	9.08229	9.08229	9.08229	9.02501
0.2	0.8	3.36345	3.36345	3.36345	3.36345	3.32012
0.4		5.02700	5.02700	5.02700	5.02700	4.95303
0.6		7.47616	7.47616	7.47616	7.47616	7.38905
0.8		11.09365	11.09365	11.09365	11.09365	11.02318

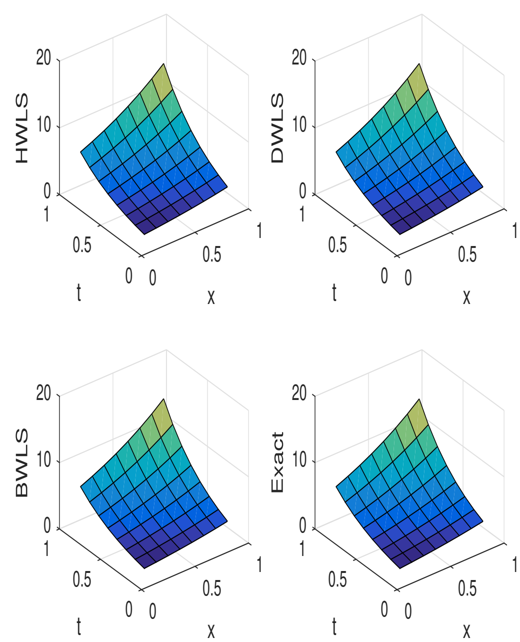

Fig. 3. Comparison of numerical solutions with exact solution of problem 4.1 for $N \times N = 8 \times 8$.

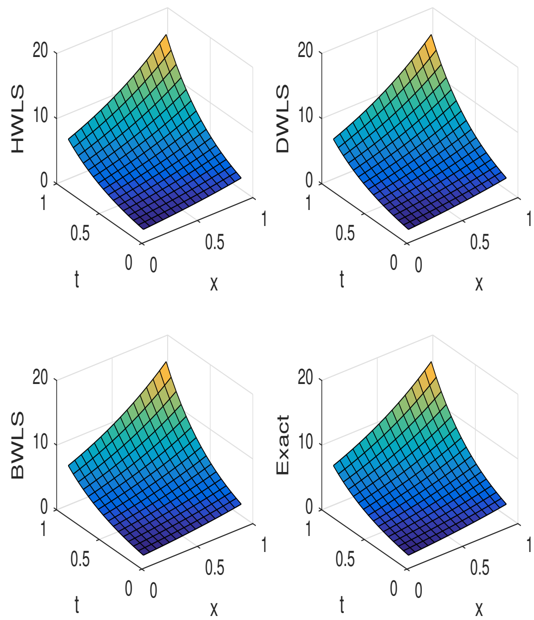

Fig. 4. Comparison of numerical solutions with exact solution of problem 4.1 for $N \times N = 16 \times 16$.

Table 2. Maximum error and CPU time (in seconds) of the methods of problem.4.1

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
4 × 4	FDM	8.7114e-02	2.8528	0.0818	29346
	HWLS	8.7114e-02	0.0006	0.0031	0.0037
	DWLS	8.7114e-02	0.0005	0.0089	0.0094
	BWLS	8.7114e-02	0.0006	0.0043	0.0049
8 × 8	FDM	4.5477e-02	5.6801	0.3622	6.0423
	HWLS	4.5477e-02	0.0006	0.0032	0.0038
	DWLS	4.5477e-02	0.0006	0.0089	0.0095
	BWLS	4.5477e-02	0.0005	0.0041	0.0046
16 × 16	FDM	2.3120e-02	3.8143	0.1284	3.9427
	HWLS	2.3120e-02	0.0007	0.0033	0.0040
	DWLS	2.3120e-02	0.0005	0.0085	0.0090
	BWLS	2.3120e-02	0.0006	0.0041	0.0047
32 × 32	FDM	1.1346e-02	5.0931	0.5715	5.6646
	HWLS	1.1346e-02	0.0007	0.0032	0.0039
	DWLS	1.1346e-02	0.0006	0.0086	0.0092
	BWLS	1.1346e-02	0.0006	0.0041	0.0047
64 × 64	FDM	5.2968e-03	4.7750	0.1355	4.9105
	HWLS	5.2968e-03	0.0008	0.0042	0.0050
	DWLS	5.2968e-03	0.0007	0.0117	0.0124
	BWLS	5.2968e-03	0.0008	0.0057	0.0065

Problem 4.2: Next, **c**onsider the non-linear Newell-Whitehead-Segel equation, $(In\ \ Eq.(1),\ a = 1,\ b = 1,\ n = 2)$

$$\text{i.e.}\quad u_t = u_{xx} + u - u^2,\ 0 \leq x \leq 1\ \&\ t > 0 \tag{15}$$

subject to the I.C.:

$$u(x,0) = \frac{1}{1 + e^{\frac{\sqrt{2}}{2}x + 1}} \tag{16}$$

and B.C.s:

$$\left.\begin{aligned} u(0,t) &= \frac{1}{\left(1 + e^{-\frac{5t}{6}}\right)^2} \\ u(1,t) &= \frac{1}{\left(1 + e^{\frac{1}{\sqrt{6}} - \frac{5t}{6}}\right)^2} \end{aligned}\right\} \tag{17}$$

Which has the exact solution $u(x,t) = \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}} - \frac{5t}{6}}\right)^2}$ [12].

Table 3 Comparison of numerical solutions with exact solution of the problem 4.2.

x	t	Numerical solution				Exact solution
		FDM	HWLS	DWLS	BWLS	
0.2	0.2	0.27216	0.27216	0.27216	0.27216	0.27169
0.4		0.25156	0.25156	0.25156	0.25156	0.25084
0.6		0.23157	0.23157	0.23157	0.23157	0.23083
0.8		0.21223	0.21223	0.21223	0.21223	0.21170
0.2	0.4	0.31700	0.31700	0.31700	0.31700	0.31651
0.4		0.29500	0.29500	0.29500	0.29500	0.29421
0.6		0.27342	0.27342	0.27342	0.27342	0.27257
0.8		0.25229	0.25229	0.25229	0.25229	0.25169
0.2	0.6	0.36404	0.36404	0.36404	0.36404	0.36371
0.4		0.34092	0.34092	0.34092	0.34092	0.34034
0.6		0.31812	0.31812	0.31812	0.31812	0.31744
0.8		0.29562	0.29562	0.29562	0.29562	0.29512
0.2	0.8	0.41254	0.41254	0.41254	0.41254	0.41244
0.4		0.38869	0.38869	0.38869	0.38869	0.38844
0.6		0.36506	0.36506	0.36506	0.36506	0.36469
0.8		0.34162	0.34162	0.34162	0.34162	0.34130

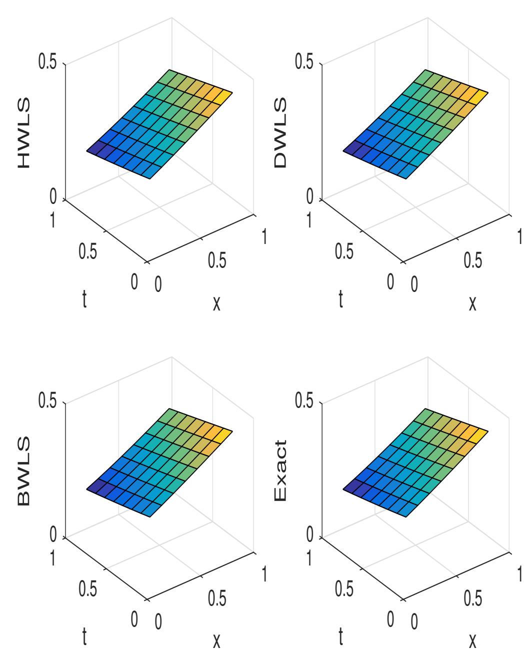

Fig. 5. Comparison of numerical solutions with exact solution of problem4.2 for $N \times N = 8 \times 8$.

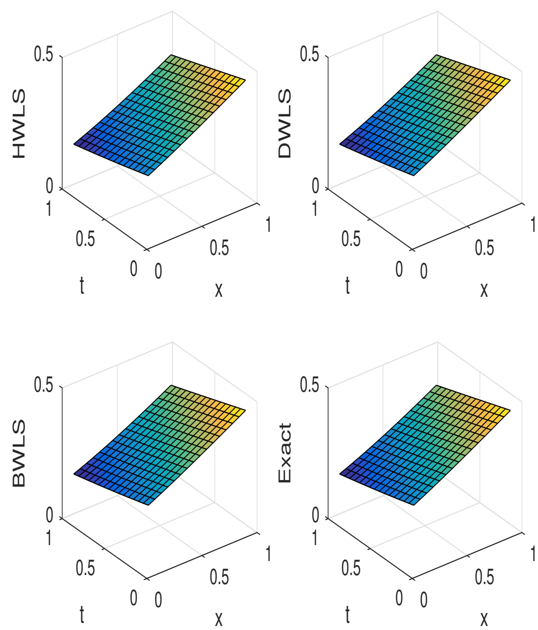

Fig. 6. Comparison of numerical solutions with exact solution of problem4.2 for $N \times N = 16 \times 16$.

Table 4 Maximum error and CPU time (in seconds) of the methods of problem 4.2.

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
4 × 4	FDM	8.5299e-04	2.6230	0.0025	2.6255
	HWLS	8.5299e-04	0.0009	0.0029	0.0038
	DWLS	8.5299e-04	0.0003	0.0096	0.0099
	BWLS	8.5299e-04	0.0006	0.0039	0.0045
8 × 8	FDM	5.4989e-04	4.0367	0.0019	4.0386
	HWLS	5.4989e-04	0.0009	0.0029	0.0038
	DWLS	5.4989e-04	0.0003	0.0096	0.0099
	BWLS	5.4989e-04	0.0003	0.0040	0.0043
16 × 16	FDM	3.0605e-04	4.0277	0.0022	4.0299
	HWLS	3.0605e-04	0.0009	0.0029	0.0038
	DWLS	3.0605e-04	0.0003	0.0099	0.0102
	BWLS	3.0605e-04	0.0003	0.0040	0.0043
32 × 32	FDM	1.6190e-04	4.2544	0.0029	4.2573
	HWLS	1.6190e-04	0.0010	0.0029	0.0039
	DWLS	1.6190e-04	0.0003	0.0096	0.0099
	BWLS	1.6190e-04	0.0003	0.0041	0.0044
64 × 64	FDM	8.3446e-05	7.7523	0.0042	7.7565
	HWLS	8.3446e-05	0.0010	0.0032	0.0042
	DWLS	8.3446e-05	0.0003	0.0104	0.0107
	BWLS	8.3446e-05	0.0003	0.0041	0.0044

5. Conclusions

In this paper, we obtained numerical solution of linear and non-linear Newell-Whitehead-Segel equations by wavelets based Lifting schemes using different wavelet filters. From the above tables 1 and 3, figures 3 to 6, we observe that the numerical solutions obtained by different Lifting schemes are agrees with the exact solution. Also, from the tables 2 and 4, the convergence of the presented schemes is observed i.e. the error decreases when the level of resolution N increases. In addition, the calculations involved in Lifting schemes are simple, straight forward and low computation cost compared to classical method i.e. FDM. Hence the presented Lifting schemes in particular HWLS and BWLS are very effective for solving non-linear partial differential equations.

References

[1] K. J. Hassan, Homotopy perturbation algorithm using Laplace transform for Newell-Whitehead-Segel equation, International Journal of Advances in Applied Mathematics and Mechanics, 2 (4), 8-12, 2015.

[2] S. A. Manaa, An approximate solution to the Newell-Whitehead-Segel equation by the Adomain decomposition method, Journal of Computer and Mathematical Sciences, 8 (1), 171-180, 2011.

[3] A. Aasaraai, Analytic solution for Newell-Whitehead-Segel equation by differential transform method, Middle-East Journal of Scientific Research, 10 (2), 270-273, 2011.

[4] G. Beylkin, R. Coif, V. Rokhlin, Fast wavelet transforms and numerical algorithms, Communications on Pure and Applied Mathematics, 44, 141-183, 1991.

[5] N. M. Bujurke, C. S. Salimath, R. B. Kudenatti, S. C. Shiralashetti, A fast wavelet-multigrid method to solve elliptic partial differential equations, Applied Mathematics and Computation, 185 (1), 667-680, 2007.

[6] N. M. Bujurke, C. S. Salimath, R. B. Kudenatti, S. C. Shiralashetti, Wavelet-multigrid analysis of squeeze film characteristics of poroelastic bearings, Journal of Computational and Applied Mathematics, 203, 237-248, 2007.
[7] S. L. Pereira, S. L. L. Verardi, S. I. Nabeta, A wavelet-based algebraic multigrid preconditioner for sparse linear systems, Applied Mathematics and Computation, 182, 1098-1107, 2006.
[8] I. Daubechies and W. Sweldens, Factoring wavelet transforms into lifting steps, Journal of Fourier Analysis and Application, 4 (3), 247-269, 1998.
[9] W. Sweldens, The lifting scheme, A custom-design construction of biorthogonal wavelets. Applied Computational Harmonic Analysis, 3 (2), 186-200, 1996.
[10] S. C. Shiralashetti, M. H. Kantli, A. B. Deshi, Wavelet Based Numerical Solution of Elasto-hydrodynamic Lubrication Problems via Lifting Scheme, American. Journal of Heat and Mass Transfer, 3 (5), 313-332, 2016.
[11] M. Mohand, M. Abdelrahim, Homotopy Perturbation Method for Solving Newell-Whitehead-Segel Equation, Advances in Theoretical and Applied Mathematics, 11 (4), 399-406, 2016.
[12] M. Zellal, K. Belghaba, Applications of Homotopy Perturbation Transform Method for Solving Newell-Whitehead-Segel Equation, General Letters in Mathematics, 3 (1), 35-46, 2017.

L. M. Angadi
Department of Mathematics, Government First Grade College, Chikodi - 591201, India
Email address: angadi.lm@gmail.com