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MODELING AND CONTROL OF AIRCRAFT PILOT STICK BASED ON ANFIS, CUCKOO SEARCH ALGORITHM AND SYSTEM IDENTIFICATION

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Abstract. This paper introduces the modeling of longitudinal motion of aircraft system using Adaptive Neuro Fuzzy Interference System (ANFIS), Cuckoo Search Algorithm (CSA) and system identification (SI). It also presents the control design of the aircraft system via applying Model Predictive Controllers (MPCs) and Proportional Integral (PI) controller to track a certain trajectory of the aircraft stick for the aircraft system. The main target for obtaining different types of system modeling is to facilitate the process of control design methodologies for longitudinal motion of aircraft. Matlab / Simulink have been used to simulate the obtained models and the designed control strategy. It is very interesting to know that flight control played an essential role in the success of ultimate for the design of earliest aircraft. In later years the automatic controls design guided in the fast development of military commercial aircraft. As a result, this paper discusses the development of modeling and control design of aircraft. To evaluate the obtained results for the resultant models and control strategy, a performance criteria including cross correlation function (XCF), Integral Time Absolute Error (ITAE), settling time and overshoot have been selected to assess the models and control methodologies. Considering the simulation results, the applying of ANFIS method has the priority for modeling the aircraft system due to obtain distinct values of evaluation criteria as compared with SI and CSA methods. In addition the MPCs as the system brain demonstrate better result in compared with Proportional Integral (PI) Controller. Finally, it can be concluded that the ANFIS modeling and MPCs have the main concern of utilizing it in the area of modeling and control of the longitudinal motion of aircraft system.

1. Introduction

Flight control of aircraft is defined as controlling the attitude and direction of an aircraft during flight by pilot. In addition, the systems of flight control are categorized into two types of flight controls such as primary and secondary control [1]-[2]. The primary control type is needed to make a safe flight control of aircraft. It contains of elevators, rudder and ailerons. However, the aim of secondary flight control is to improve the performance characteristics of aircraft or to mitigate the over loading control and include devices of high lift [1]-[2]. Flight dynamics is defined as the air vehicle direction science and the control around three dimensions (pitch, roll and yaw). It is known that the main aim of control systems in flight dynamics is to regulate the vehicle orientation around its cg [1]-[2]. The control system involves control surfaces.

In case of there is deflection in the control surface, the control system will create a moment around the cg. As a result, the aircraft will rotate in pitch, roll, and yaw. If there is an applying force at a distance aft or forward to the cg, it will produce a pitching moment which makes a pitching up or down aircraft [3]. One of the main difficult problems that facing the scientific research when the research aims to design and implement an acceptable control system for a specific application are the procedure of how to construct a simulation model to test the proposed control design. As a result the simulation of the model is the beginning step for any control methodology design. For these reasons, this article research focuses on obtaining different types of a simulation model and control methodologies for the aircraft system and verified the output with the original system.

The ANFIS method is used to model the estimation of quality in used refining palm process oil [4]. In addition, the process of refining of palm oil is defined as the separating of the undesirable composition from palm oil crude to create a filtered, deodorized and bleached palm oil [4]. The electro hydraulic device system is implemented and modeled based on system identification as discussed in [5]. Moreover in [5], the system model is described as transfer function model that is constructed using neural networks and fuzzy system.

Key words and phrases. Longitudinal motion of aircraft, ANFIS, ITAE, MPC, CSA, System Identification.

In Ref. [6] the authors investigated ANFIS modeling, analysis and parameters optimization for micro-EDM. Moreover, the researchers studied a new approach that focused on full factorial architecture for ANFIS modeling in [7]. In addition, the house model with thermal temperature control and system identification has been presented in [8]. The strategy of MPC that tuned by CSA has been used for temperature control and the system identification is utilized to build a system model as discussed in [8]. Furthermore, the fuzzy logic control is used to control the longitudinal motion of aircraft as demonstrated in [9]. A solution of structural optimization problems has been prepared via CSA that is considered as a metaheuristic approach [10]. Artificial neural networks have been used to build a model for aircraft system with a given flight system data in [11]. The technique of ANFIS modeling displayed the capability to build a model for nonlinear system like Electro-Hydraulic Actuator system through physical modeling and FCM gap statistic with high accuracy as given in [12]. Moreover in [13], the initial Fuzzy Inference System (FIS) is required before training of ANFIS parameters. Different methods of system Identification for m-ary frequency shift keying signals via applying discrete wavelet transformation and ANFIS technique [13]. In Ref. [14], implementation and control of a nonlinear experimental model of hydraulic servo system (HSS) have been discussed. The obtained model is based on physical laws and system identification [14]. The parameters of the suggested controllers have been tuned via particle swarm optimization technique as presented in [14]. The system identification is used to model an aircraft and rotorcraft system with different engineering method that is applied to flight test example as described in [15].

2. System Modelling

The second law of Newton can be used to obtain the mathematical representation that governs the equation of motion for flight vehicle. As a result, the equation of the force can be represented mathematically as follows [11], [16].

$$\sum F = \frac{d}{dt}(mv) \tag{1}$$

Where F stands for the components of the force (F_x, F_y, and F_z) on three given axes ($x, y, andz$), m represents the mass, v is a symbol for contributions of velocity from both rates of rotational (p, q, and r) and linear (u, v, and w) about x, y, and z-axes, respectively. Moreover, the components of force are created by contributions because of the sum of propulsive, aerodynamic, and the force of gravitational affecting on the airplane. The equation of moment can be stated as given in Equation (2) [9], [11], [16],

$$\sum M = \frac{d}{dt}(H) \tag{2}$$

Where, M symbolizes to components of moment for instance L, M, and N on the relevant three axes x, y, and z. As well H corresponds to momentum components moment including H_x, H_y, and H_z, alongside of the three axes x, y, and z, respectively. The moments and forces of aerodynamic can be written in a function of all the variables of motion. As a result, all the governing equations of motion can be expressed as in Equations (3), (4) and (5) [1]-[3], [16].

$$\left[\frac{d}{dt} - X_u\right] u + g_0 \cos\theta_0 - X_w w = X_{\delta_e}\delta_e + X_{\delta_T}\delta_T \tag{3}$$

$$-Z_u u + \left[(1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right] w - [u_0 + Z_q]\, q + g_0 \sin\theta_0 = Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T \tag{4}$$

$$-M_u u - \left[(M_{\dot{w}})\frac{d}{dt} - M_w\right] w + \left[\frac{d}{dt} - M_q\right] q = M_{\delta_e} + M_{\delta_T}\delta_T \tag{5}$$

Where, X_w, Z_w, M_w and $M_{\dot{w}}$ are described as derivatives of stability, that are assessed at the condition of reference flight. In addition, the variables of control including δ_T and δ_e stand for variations from trim in the thrust (throttle) and elevator settings. As well $X_{\delta_e}, Z_{\delta_e}, M_{\delta_e}$ represents to settings of elevator for X-force, Z-force and moment due to pitching. The variables $X_{\delta_T}, Z_{\delta_T}, M_{\delta_T}$ refer to settings of throttle for X-force, Z-force and moment due to pitching. The derivative of the described variables can be displayed by a dot above these variables in Equations (3), (4), and (5) [16]. Fig. 1 depicts the components of moments, force, and velocity components in a coordination of body fixed.

Figure 1. Moments, force, and velocity components

3. System Identification

System identification is defined as a system that can build a gray or black model for a dynamic system via the measuring of a collected data for the input and the output (I/P) [8], [17]. As a result, the estimated mathematical model is known as a representation between the system variables of the input and the output. As well, the system dynamic models are described by difference / differential mathematical equations, continuous /discrete transfer functions and state space representations [8],[17].

One of the types of system identification is called as ARX model. The ARX states to an auto-regressive models with exogenous inputs. It characterized as a linear parametric type of model that expressed as a structure of a linear difference equation as given in Equation (6). The structure of ARX find a relation between the present output $y(t)$ to a limited number of past outputs $y(t-k)$ and inputs $u(t-k)$. The ARX model structure could be presented in time domain as follows:

$$y(t) + a_1 y(t-1) + \dots\dots\dots + a_{na} y(t-na) = b_1 u\,(t-n_k) + \dots\dots + b_{nb} u\,(t-n_b-n_k+1) + e(t) \tag{6}$$

Where, n_a and n_b are expression of the orders of the model, n_a and represents to poles and zeros number and n_k is the delay or samples number. In addition, $u(t)$, and $y(t)$ refer to input and output at time t and $e(t)$ is the noise.

In this article, the best fit criteria (BF) (see equation (7)) that measure how the model describes the process much better compared to the output mean [8], [17].

$$\text{BestFit } = \left(1 - \frac{|y-\hat{y}|}{|y-\bar{y}|}\right) * 100 \tag{7}$$

Where the measured output is symbolized by $(y), \hat{y}$ demonstrates the forecasted model output, $\bar{y}$ represents the mean of y.

4. ANFIS Modelling

A fuzzy inference system (FIS) is defined as the procedure of applying the fuzzy sets theory and rules to map a presented input to an output [18]-[20]. The FIS performance based on two important parameters including member ship functions (MFs) and fuzzy rules identification of membership functions (MFs) and the fuzzy rules that is tuned according to the studied applications. It is known that in many cases there are a difficultly to convert the human knowledge to a tuned fuzzy rules and MFs [18]-[20]. As a result, artificial neural network (ANN) was introduced to overcome the mention limitation via presenting a fuzzy rules and MFs automatically [18]. The parameters of ANFIS system are trained by ANN learning algorithm. It is a multilayer feed forward based on using Sugeno type [19]. The number if input and output of ANFIS system are assumed to be two inputs and one output. For more details about ANFIS Layers (see Ref. [20]).

5. Cuckoo Search Algorithm

Xing-she yang and Susah developed a firstly Cuckoo Search Algorithm (CSA) in 2009 [8], [21]. The cuckoos breeding behavior is used to inspire the cuckoo algorithm. CSA is defined as a method of meta-heuristic search [8], [21]. In addition there is an only one parameter requires to be examined. The Levy flights method is used for the cuckoo search by means of CSA will get each optimum in the space and will compare with the others.

The generation process of new solutions x^{t+1} for a cuckoo i based on using levy flight method. The govern equation for a new solution is given as follow [8], [21]:

$$\mathrm{x_i^{(t+1)}} = \mathrm{x_i^{(t)}} + \alpha \oplus \mathrm{L\tilde{A}©VY}(\lambda) \tag{8}$$

Where, α represents the size of step ($\alpha \geq 0$). The value is chosen to be associated with the discussed problem scales. The value of alpha is chosen according to the relation of alpha as equal to as $\alpha = O(L/10)$ where L stands for the characteristic scale of the problem the product $\oplus$ represents multiplication of entry wise. The used product in CSA is resembled to those in PSO. On the other hand the walking random via Lévy flight is more efficient in the case of discovering the search space as its length of step is more longer in the long run.
Actually, the Lévy flight demonstrates a random walk whose the length of random step is obtained from the distribution of Lévy. As given in the following equation.

$$\mathrm{L\tilde{A}©vy\ u} = \mathrm{t}^{-\lambda}, (1 < \lambda \leq 3) \tag{9}$$

In CSA the new solution must be produced via Lévy around the obtained best solution as far as, this will accelerate the local search. In CSA the fitness function is given as the following Equations (10 - 11)

$$\text{Fitness function } = \frac{1}{\text{ITAE}} \tag{10}$$

Where, ITAE is integral time absolute error

$$\text{ITAE} = \int_0^\infty t|e(t)|dt \tag{11}$$

Where t denotes to time and $e(t)$ is the error

6. Model Predictive Control

The strategies of modern control are considered as the core of industrial application that can improve the performance of the system and enhance the rate of production. The MPC is one of the promising algorithms of advanced control [8]. It may appropriate for most of industrial process due to the MPCâs capability to control the system within constraints [22]. MPC have the ability of capturing the dynamic models for the industrial process, most often models based on experimental that may be structured by system identification. On the other hand, Most of field engineers are not recognizable with the modern control techniques structure of conceptual, the behavior between control system actions and the parameter settings or tuning for this control system. As a result, it is still a limited used and implemented in the industrial field [8], [22].

The operation of MPC is based on a mathematical equation called cost function and symbolized as $(J(k))$. The equation of cost function and the constraints is depicted as follow [8], [22]:

$$\mathrm{J(k)} = \sum_{i=1}^{\mathrm{p}} \mathrm{Q.}\ [\mathrm{\hat{y}(k+i \mid k) - r(k+i \mid k)}]^2 + \sum_{\mathrm{i=0}}^{\mathrm{M-1}} \mathrm{R} \cdot [\Delta \mathrm{u(k+i \mid k)}]^2 \tag{12}$$

Subject to

$$y_{\min} \leq \hat{y}(k+i \mid k) \leq y_{\max} \tag{13}$$

$$u_{\min} \leq u(k+i \mid k) \leq u_{\max} \tag{14}$$

$$\Delta u_{\min} \leq \Delta u(k+i \mid k) \leq \Delta u_{\max} \tag{15}$$

Where, P and M are prediction and control horizon respectively, k is a symbol of discrete time, i is index during P interval, Q and R refer to weights of output error and changing in manipulated variable respectively, $\mathbf{\hat{y}}(\mathrm{k+i|k})$ and $r(k+i|k)$ stand for forecasted output and reference at time $k+i$ respectively, $u(k+i|k)$ and $\Delta u(k+i|k)$ are the forecasted best manipulated variable and changing rate of predicted manipulated variable at time $k+i$ respectively.

7. Results and Discussions

This section will be divided into two part. The first part contains the results and discussion for the three modeling techniques, although the second part introduces the discussion of control design for tracking control. The two parts are given below as follows.

7.1. **Modelling Techniques.** The aircraft system has been configured to work in closed loop condition with proportional and integral parameters are -1.746 and -3.864 respectively. The modeling based on system identification process has been obtained first by acquiring a set of output response due to excitation signal data Fig. 2 displays the used excitation signal for system identification. The identified model in this research is chosen to be as a transfer function type. Considering Fig. 2, the used excitation signal to excite the system dynamics and can collect most of these dynamics is a multistep signal with variable step amplitude and variable step frequency. A number of samples equal to 1400 samples are utilized in Matlab/ SI toolbox. To build an accurate model, the first 700 samples are applied to SI toolbox to give the identified model while the other 700 samples are used for the validation of the model. Equation (16) demonstrates the identified model. The evaluation criteria including BF and XCF have a value about 94.52% and 97.02% inch respectively. The aircraft response based on SI model is given in Fig. 3

$$\frac{Y(s)}{X(s)} = \frac{0.2333s^3 + 34.91s^2 + 203.3s + 157.6}{s^5 + 7.562s^4 + 55.43s^3 + 161.4s^2 + 313.5s + 157} \tag{16}$$

Where $X(s)$ and $Y(s)$ stand for pilot stick input and system response respectively.

A structure of a transfer function with third order system has been suggested to represent the system model. The coefficient and optimal parameters of numerator and dominator for the model will be tuned and searched by CSA as an optimization technique. The suggested structural model is depicted below in Equation (17). The optimal parameters that tuned by CSA are decided to be selected according to a fitness function as given in Equation (10). The minimum value of ITAE will produce optimal parameters. The range of the coefficient is given to the cuckoo algorithm via our experience about the system. The range of the parameters in Equation (17) are: $a_1 = [190250], a_2 = [150200], b_1 = [5070], b_2 = [250300]$, and $b_3 = [150200]$. The optimized aircraft model based on CSA is depicted in Equation (18).

$$\frac{Y(s)}{X(s)} = \frac{a_1 * s + a_2}{b_1 * s^2 + b_2 * s + b_3} \tag{17}$$

$$\frac{Y(s)}{X(s)} = \frac{206s + 162}{56s^2 + 290s + 154} \tag{18}$$

The simulation result between proposed CSA model and the original aircraft longitudinal motion model is presented in Fig.4. The validation result of CSA model with aircraft system response has cross correlation (XCF) percentage at 98.37% and ITAE of 79.8.

The ANFIS model is structured based on two inputs and one output. The two inputs are selected to be a delayed one sample of stick response and a delayed one sample of alpha (angle of attack). This idea was taken from the mathematical equations that govern the longitudinal motion of aircraft. As a results there is a needing to acquire data sets of $Y(K), Y(k-1)$ and alpha $(k-1)$.

The ANFIS is trained using the hybrid learning algorithm and the type of MF chosen is the trapezoidal - mf type membership functions. The validation process of the obtained model is performed by make a comparison between the ANFIS model and the actual response of the system. The test of the model accuracy is developed by XCF and ITAE between ANFIS model and original system. The ANFIS modeling of aircraft system presents values about 99.92% and 0.024932 inch for XCF test and average square error respectively. Fig. 5 depicts the simulation result between suggested ANFIS type model and the original model of aircraft system depended on a set of validation data.

Finally, the evaluation criteria (XCF) values for SI, CSA and ANFIS are 97.02%, 98.73%, and 99.92% respectively. As a result, the simulation results indicate that ANFIS model has performed a better job modeling of aircraft system compare to SI and CSA.

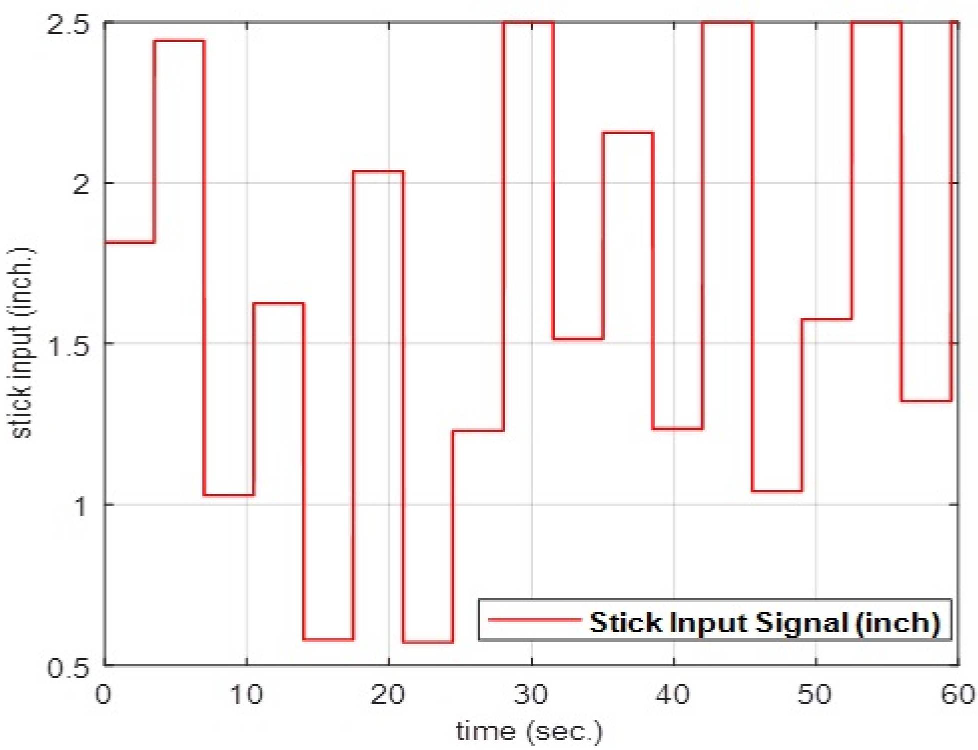

Figure 2. Excitation signal for aircraft

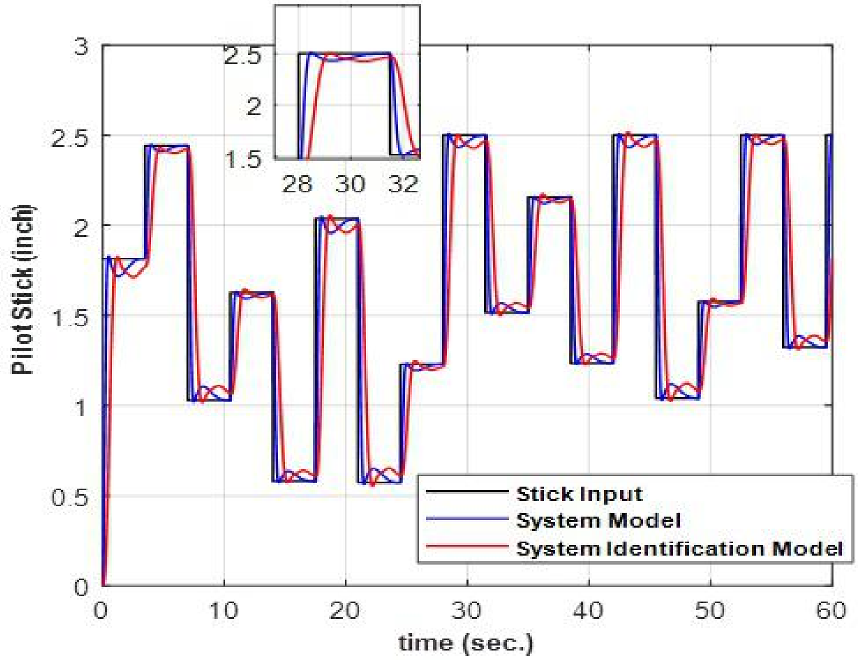

FIGURE 3. System modeling based on SI technique

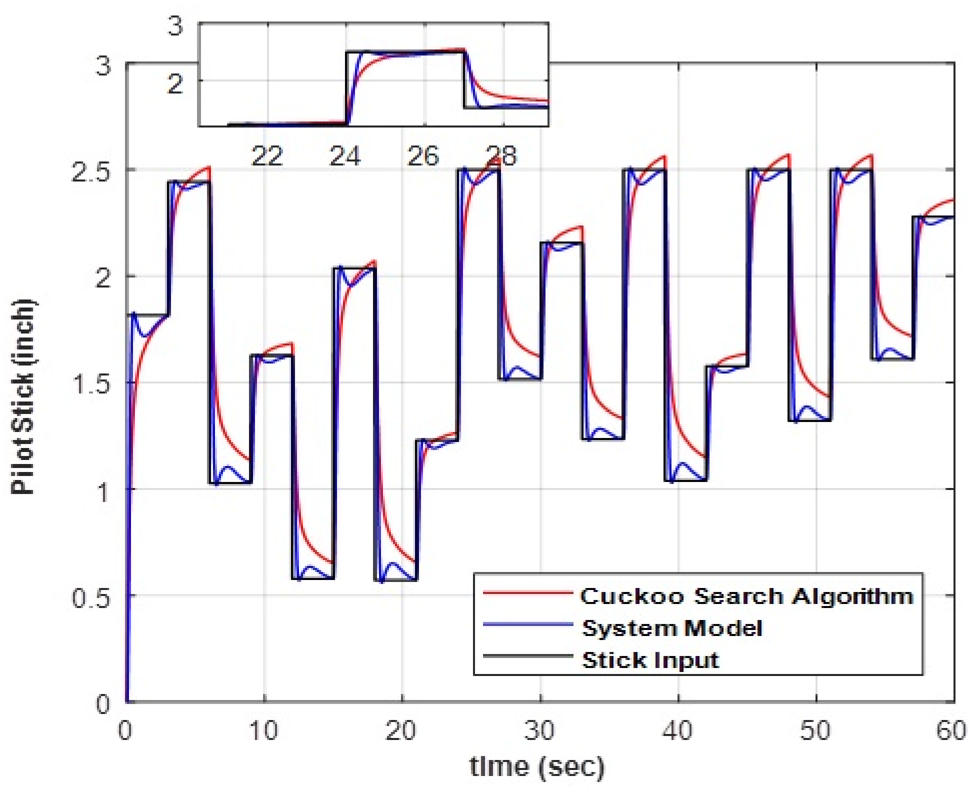

FIGURE 4. System modeling based on CSA technique

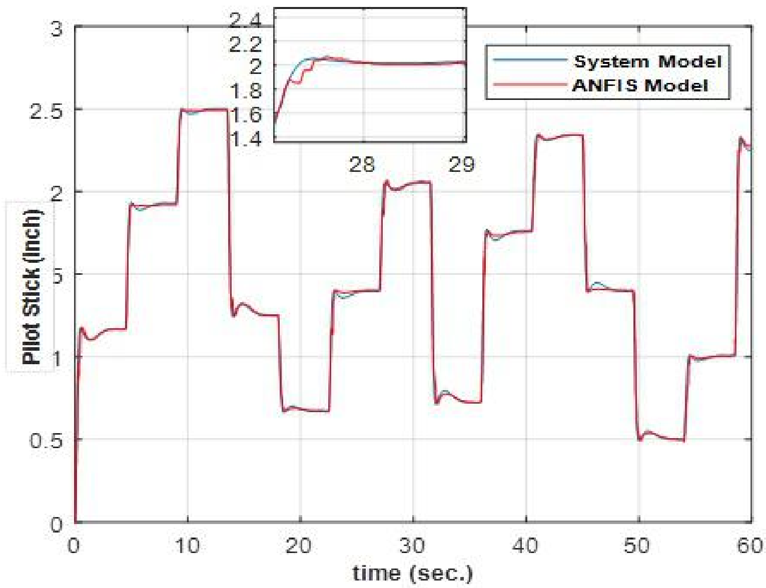

FIGURE 5. System modeling based on ANFIS technique

7.2. **MPC Control Design.** MPC and PI controllers have been used to track a certain trajectory for the aircraft pilot stick as introduced in Figs. 6 and 7. A comparative study between the output responses for the two control strategy is performed using settling time and rise time in seconds, and percentage of maximum stick motion overshoot. The tuning parameters of MPC and PI controllers are given in Table 1. Figs. 6 and 7 shows the result of the step and square input signals between suggested MPC and PI controllers. MPC controller has a best settling time, rise time and % of max. Overshoot at 0.44 sec, 0.23 sec, and 2% respectively compare to 3 sec, 0.8 sec, and 40% in PI. As viewed in zoomed figures in Figs. 6 and 7. PI fails to track the command signal of pilot stick during the changing of direction of stick motion as demonstrated in Figs. 6 and 7. However, the MPC showed a better tracking in the simulation zoomed figures. As a result, MPC indicated a better tracking for the longitudinal motion of aircraft system compare to PI control methodology.

TABLE 1. PARAMETERS OF USED MPC AND PI

MPC	
Sample Time (seconds)	0.01
Prediction Horizon (P)	8
Control Horizon (M)	1
Weight (Q)	0
Rate Weight (R)	0.1
Output weight	1
PI	
Proportional Gain (Kp)	-1.746
Integral Gain (Ki)	-3.864

8. CONCLUSION

Aircraft Flight control is the ability of the system to control the aircraft direction and its attitude during flight via pilot stick. The longitudinal motion of an aircraft was taken as a part of flight control to introduce different design of aircraft models and control methodologies. The aircraft pilot stick is considered to be the main part that responsible for the flight control of longitudinal motion of the aircraft. The artificial intelligence techniques such as ANFIS and CSA are adopted to model the longitudinal motion system as well as system identification toolbox. For the part of control, the PI and MPC are designed then evaluate based on a defined performance criteria to choose the suitable and robust controller type. The simulation results presented a distinct cross correlation test and ITAE test for the system modeling by ANFIS over SI and CSA. In addition, the MPC forced the aircraft system to track the command pilot stick efficiently in case of assessed it with PI controller. This research proved that the efficient use of a promising ANFIS technique and MPC strategy for modeling and control respectively the longitudinal motion of aircraft system.

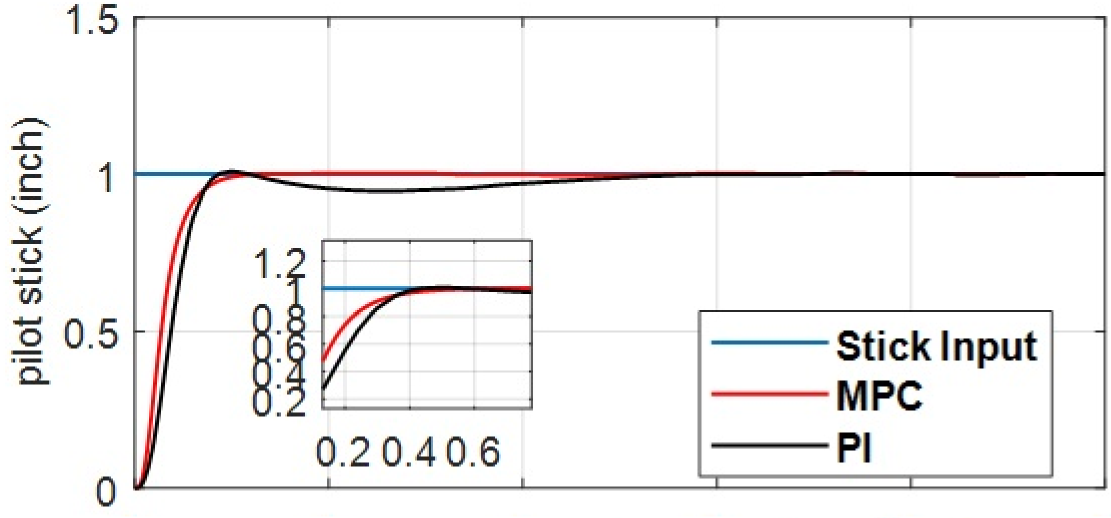

Fig. 6. Step response for the tracking of aircraft pilot stick

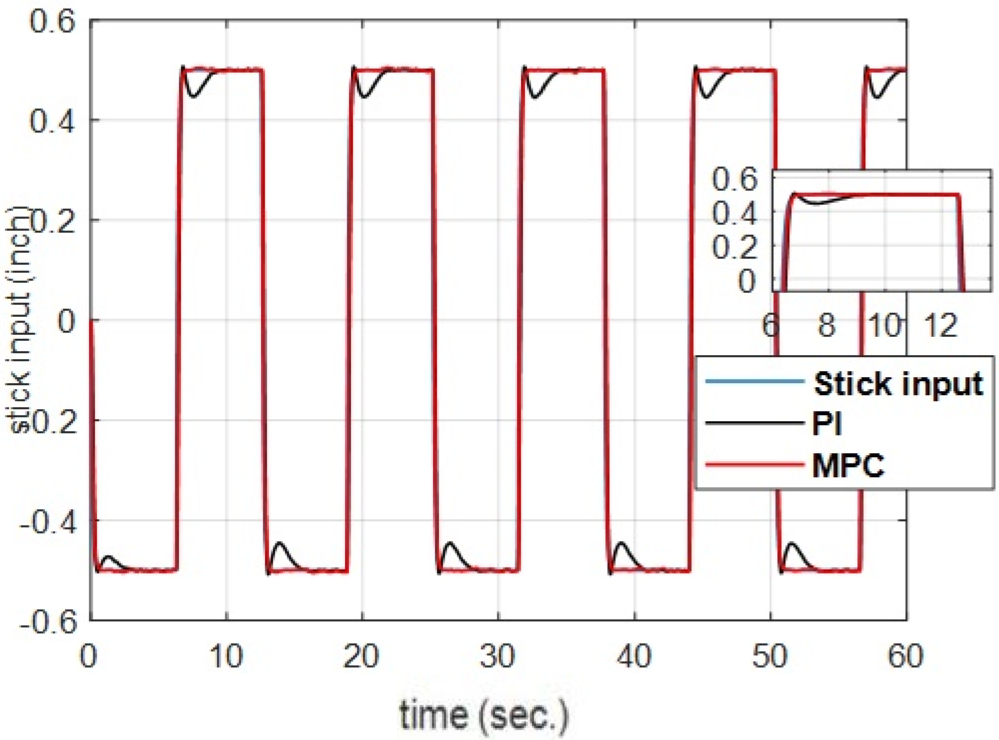

Figure 7. Square wave response for the tracking of aircraft pilot stick

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