Journal of Fractional Calculus and Applications
Vol. 12(3). No. 6, pp. 1-10
1st. Inter. E-Conf. in Math. Sciences and Fractional Calculus(ICMSFC Feb 2021).
ISSN: 2090-5858.
http://math-frac.org/Journals/JFCA/

FRACTIONAL CALCULUS OF REAL AND COMPLEX VARIABLES AND ITS APPLICATIONS

RABHA W. IBRAHIM

Abstract. In this note, we give a presentation of fractional calculus and its applications. The extension of the fractional differential and integral fractional operators is investigated in the sequel. Most recent applications in physics, mathematical biology and economy are included.

Factional calculus is documented since the usual calculus with the first written reference dated in September 1695 in letter from Leibniz to L'Hospital. The α derivative of a function $f(x)$ at a point f is a local property only when α is an integer; this is not the situation for fractional power derivatives. In other words, a fractional derivative of a function $f(x)$ at x depends on all values of f. Therefore, it is probable that the fractional derivative operation includes some kind of boundary conditions, connecting information on the function further out. In 1832, Bernhard Riemann and Joseph Liouville formulated the first fractional differential and integral operators The Riemann-Liouville integral is defined by

$$I^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x f(t)(x-t)^{\alpha-1}\, dt, \quad \alpha > 0$$

corresponding to the fractional differential operator by defining

$$D_x^{\alpha} f(x) = \begin{cases} \frac{d^{\lceil\alpha\rceil}}{dx^{\lceil\alpha\rceil}} I^{\lceil\alpha\rceil-\alpha} f(x) & \alpha > 0 \\ f(x) & \alpha = 0 \;; \\ I^{-\alpha} f(x) & \alpha < 0. \end{cases}$$

$$D_x^{\alpha} f(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_0^x \frac{f(t)}{(x-t)^{\alpha}} dt.$$

The In 1967, Caputo introduced the following fractional derivative

$$D_x^{\alpha} f(x) = \frac{1}{\Gamma(1-\alpha)} \int_0^x \frac{f'(t)}{(x-t)^{\alpha}} dt, \quad \alpha \in [0,1).$$

1991 *Mathematics Subject Classification.* 30C45.
Key words and phrases. Fractional calculus, conformable fractional calculus, open unit disk, analytic function, univalent function .
Submitted Feb. 13, 2021.

The earlier application of fractional calculus is appeared when Heaviside, 1890 presented the practical usage of fractional differential operators in electrical transmission line analysis circa. Then the theory and applications of fractional calculus expanded greatly over the 20th century, and many researchers have formulated definitions for fractional derivatives and integrals.

Currently, the fractional calculus has an extensive range of applications in science, technology and social studies. In all those applications, the fractional order plays an important role to describe the modeling structures. Furthermore, the fractional calculus plays an important role even in the complex analysis specifically, the geometric function theory and univalent function theory. Therefore, it permits us to use an improved explanation of certain real-world phenomena. Based on this fact, the fractional calculus (real and complex) is global as well as whole real world around us is fractional, not integer one. Due to this reason, it is so critical consider almost all schemes as the fractional order structures.

1. Complex Fractional Calculus

1.1. **1D-fractional order differential and integral operators.** In complex analysis, there is a major branch called the geometric function theory (GFT). This theory deals with the geometric representation of a conformal function, which is a function that locally reserves angles, but not essentially lengths. In GFT, there is a class of differential operators. These operators play a substantial character to generalize some classes of analytic functions. For example the Salagean differential operator (1981) [1] of a normalized function $f(z), z \in U$ $(f(0) = f'(0) - 1{=}0)$, where U is the open unit disk

$$Df(z) = zf^{'}(z), ..., D^n f(z) = D(D^{n-1})f(z) \quad z \in U \tag{1}$$

is generalized to the 1D-fractional Salagean differential operator (2009) [2]

$$D^{n+\alpha} f(z) = \frac{d^n}{dz^n} D^\alpha f(z), \quad z \in U, \tag{2}$$

where D^α is indicated the Srivastava-Owa differential operator (1989) [3]

$$D^\alpha f(z) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\alpha} d\zeta, \quad 0 \le \alpha < 1;$$

corresponding to the fractional integral operator

$$I^\alpha f(z) = \frac{1}{\Gamma(\alpha)} \int_0^z \frac{f(\zeta)}{(z-\zeta)^{1-\alpha}} d\zeta, \quad \alpha > 0.$$

Ibrahim and Darus 2008 established the existence and uniqueness of classes of fractional differential equations in a complex domain such as [4, 5]

$$D^\alpha u(z) = f(z, u), \quad z \in U.$$

1.2. **2D-fractional order differential and integral operators.** Ibrahim (2011)[6] formulated 2D-fractional differential and integral operators as follows:

$$D^{\alpha,\mu} f(z) = \frac{(\mu+1)^\alpha}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{\zeta^\mu f(\zeta)}{(z^{\mu+1} - \zeta^{\mu+1})^\alpha} d\zeta, \quad 0 \le \alpha < 1;$$

corresponding to the fractional integral operator

$$I^{\alpha,\mu} f(z) = \frac{(\mu+1)^{1-\alpha}}{\Gamma(\alpha)} \int_0^z \frac{\zeta^\mu f(\zeta)}{(z^{\mu+1} - \zeta^{\mu+1})^{1-\alpha}} d\zeta, \quad \alpha > 0, \mu > -1.$$

The 2D-fractional Salagean differential operator becomes

$$D^{n+\alpha,\mu} f(z) = \frac{d^n}{dz^n} D^{\alpha,\mu} f(z), \quad z \in U. \tag{3}$$

Tremblay [7] studied a fractional calculus operator defined in terms of the Riemann-Liouville fractional differential operator. Ibrahim and Jahangiri [8] extended this operator utilizing the Srivastava-Owa differential operator as follows:

$$T^{\alpha,\beta} f(z) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} z^{1-\beta} D^{\alpha-\beta} z^{\alpha-1} f(z), \quad z \in U.$$

Therefore, we obtain another type of the 2D-fractional Salagean differential operator becomes

$$T^{n+\alpha,n+\beta} f(z) = \frac{d^n}{dz^n} T^{\alpha,\beta} f(z), \quad z \in U. \tag{4}$$

2. Complex Conformable Calculus

Anderson and Ulness (2015) [9] formulated a new conformable operator as follows:

$$\Delta^\alpha \phi(x) = \kappa_1(\alpha, x)\phi(x) + \kappa_0(\alpha, x)\phi'(x).$$

Again Ibrahim and Jahangiri (2019) [10] extended this operator to the open unit disk utilizing the normalized function $f(z)$as follows:

$$\Delta^\alpha f(z) = \left(\frac{\kappa_1(\alpha, z)}{\kappa_0(\alpha, z) + \kappa_1(\alpha, z)} \right) (f(z)) + \left(\frac{\kappa_0(\alpha, z)}{\kappa_0(\alpha, z) + \kappa_1(\alpha, z)} \right) (zf'(z)), \tag{5}$$

$$\left(0 < \alpha < 1, z \in U, \lim_{\alpha \to 1} \kappa_1 = \lim_{\alpha \to 0} \kappa_0 = 0, \lim_{\alpha \to 0} \kappa_1 = \lim_{\alpha \to 1} \kappa_0 = 1\right).$$

Clearly, Operator (5) indicates the Conformable Salagean operator, where when $\alpha \to 1$ we obtain the ordinary Salagean operator.

3. Examples

The class of Koebe functions is the most important class of analytic, normalized functions in the open unit disk. These functions are considered as extreme functions for well known subclasses of analytic functions such as the subcless of starlike functions $f(z) = z/(1-z)^2$ and the convex functions $g(z) = z/(1-z)$. We shall consider these two types in our examples (see Figs.1 and 2).

4. Resent applications

The applications of fractional calculus (real and complex) are extended to many fields specifically the following subjects (see Fig.3).

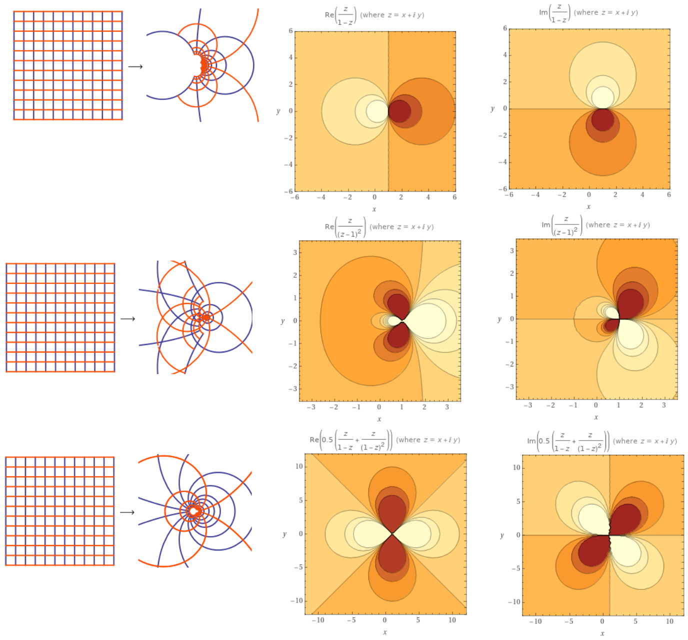

FIGURE 1. The plot of $g(z) = z/(1-z)$ (the first row); the plot of $g(z)$ under the Salagean operator (the second row); the plot of $g(z)$ under the complex conformable operator for $\alpha = 1/2$ and $\kappa_0 = \alpha z^{1-\alpha}$ and $\kappa_1 = (1-\alpha)z^{\alpha}$ (the third row)

4.1. **Physics.** Under the quantum theory, different classes of Briot-Bouquet differential equations have been generalized ([11]-[15]). This category of ODE is an association of ODE whose outcomes are formulas in the complex plane. Existence and uniqueness theorems include the utility of majors and minors (or subordination and superordination concepts). The main structure of this category is given by the formula

$$\beta f(\chi) + (1-\beta)\frac{\chi\,(f(\chi))'}{f(\chi)} = h(\chi), \quad h(0) = f(0),\, \beta \in [0,1],$$

which the term $(f(\chi))'$ is modified by using fractional calculus differential operator, Conformable differential operator and q-calculus differential operator.

4.2. **Soft computing.**

(I) This field contains some studies of multi-agent systems in cloud computing theory and cryptographic technique ([16]-[20]). The multi-agent system for the first time is given by a class of ODEs is in 2007 [21] as follows

$$\chi_i^{'}(t) = \sum_{j=1}^{n}(\chi_j(t) - \chi_i(t)) + b_i(t),$$

where χ_i is the position of the agent i at time t (integrator agent). This system is generalized in different ways by using the RL-differential operator

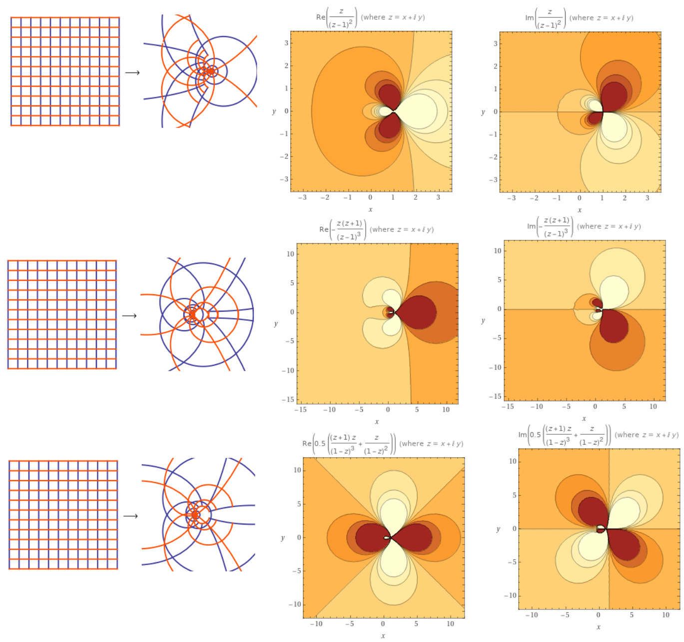

FIGURE 2. The plot of $f(z) = z/(1-z)^2$ (the first row); the plot of $f(z)$ under the Salagean operator (the second row); the plot of $f(z)$ under the complex conformable operator for $\alpha = 1/2$ and $\kappa_0 = \alpha z^{1-\alpha}$ and $\kappa_1 = (1-\alpha)z^{\alpha}$ (the third row)

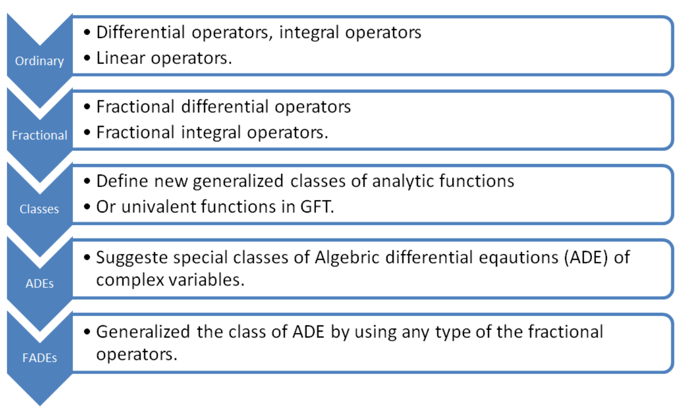

FIGURE 3. The research stages

and Caputo differential operator [22, 23] as follows:

$$\begin{aligned} D^{\alpha}\chi_k(t) &= \Phi_k(t, \chi, u, \mu), \quad k = 1, \ldots \\ &= \sum_{i=1}^{\infty} \alpha_{ii,k}(\chi_i(t))^2 + 2\sum_{i<j}^{\infty} \alpha_{ij,k}\chi_i(t)\chi_j(t) + B_k(t), \end{aligned} \tag{6}$$

$$\Big(\chi_1 = \chi_0, \quad \chi_0 \in \mathbb{R}\Big).$$

And the iterative multi-agent system

$$D^{\alpha}\chi_i(t) = \Phi_i\Big(t^{\alpha_0}, t^{\alpha_1}\chi_i^{[1]}(t), t^{\alpha_2}\chi_{i+1}^{[2]}(t), ..., t^{\alpha_n}\chi_{i+n-1}^{[n]}(t)\Big) \tag{7}$$

initialed to the value

$$\chi(t_0) = c, \quad c \in (0, \infty),$$

where $\chi^{[j]}(t) := \chi(\chi^{[j-1]}(t))$.

(II) Fractional Chaotic Maps have practiced in [24] using the fractional Chebyshev chaotic transforms and the concept of local fractional calculus (fractal)

$$T_n^{\alpha}(\tau) = \frac{2^{\alpha}}{\Gamma(1+\alpha)} T_n(\tau),$$

where $T(\tau)$ indicates the Chebyshev polynomial. Note that RL-Fractal is given by the operator

$$D^{(\alpha)} f(x) = \lim_{x \to x_0} \frac{\Delta^{\alpha}(f(x) - f(x_0))}{(x - x_0)^{\alpha}},$$

where $\Delta^{\alpha}(f(x) - f(x_0)) = \Gamma(1+\alpha)(f(x) - f(x_0))$.

4.3. **Biology.**

(I) Artificial biological system (ABS) is formulated as follows [25]

$$\frac{d\chi(t)}{dt} = \nu\phi(\chi), \tag{8}$$

where $\chi(t)$ indicates the concentrations of each molecule type during time t, ν (fixed constant for all states) represents the stoichiometry value of reaction networks and ϕ refers to the rate of change of the concentration of each type.

Recently, the concept of conformable calculus (CC) is used to generalize the ABS [26] as follows:

$$\begin{aligned} \mathcal{D}^{\beta}\chi(t) &= \nu_1(\beta, t)\chi(t) + \nu_0(\beta, t)\chi'(t) \\ &= \nu_1(\beta, t)\chi(t) + \nu_0(\beta, t)\Phi(t, \chi). \end{aligned} \tag{9}$$

(II) Furthermore, conformal geometry of the turtle shell is modeled by using the conformal analytic normalized function $\varphi(z), z \in U$ [27]

$$\begin{aligned} \Delta_{\beta}\varphi(z) &= (\frac{\beta}{\overline{\beta}})(z\varphi'(z)) - (1 - \frac{\beta}{\overline{\beta}})(z\varphi'(-z)) \\ \Delta_{\beta}^{\alpha}\varphi(z) &= \Delta_{\beta}(\Delta_{\beta}^{\alpha-1}\varphi(z)) \\ &= z + \sum_{n=2}^{\infty} \left(n \left(\frac{\beta}{\overline{\beta}} - (1 - \frac{\beta}{\overline{\beta}})(-1)^n \right) \right)^{\alpha} \varphi_n z^n, \alpha > 1, \end{aligned} \tag{10}$$

where β is a complex number. Fig.4 shows the cases of the shell.

(III) Another application of the conformal analytic function is appeared to establish a new modeling system of arched foot testing, using Eq.(10) (see [29]). We considered the roots of the Koebe function $z/(1-z)^{\sigma}$ under the operator (10).

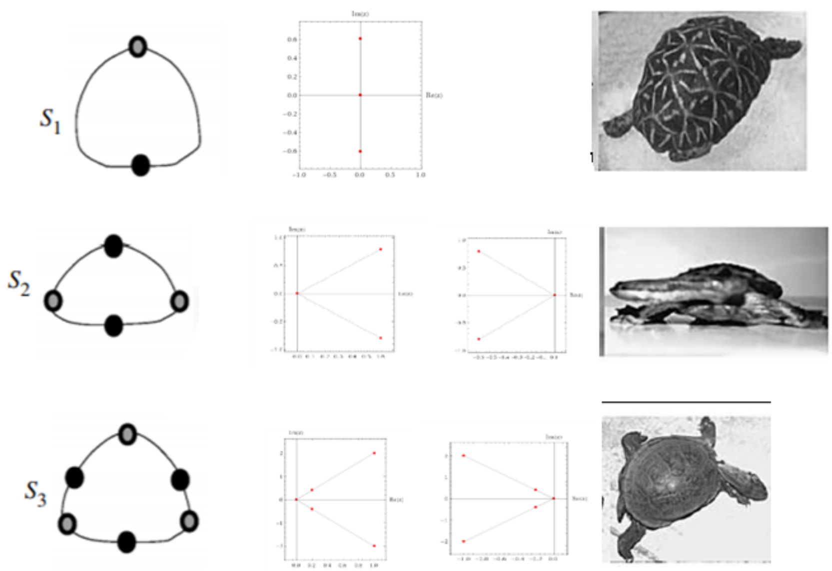

FIGURE 4. Solutions of (10), when $f_1(z) = z/(1-z), f_2(z) = z/(1-z)^2, f_3(z) = z/(1-z)^3$. The first column is given by Domokos and Varkonyi, 2007 [28]

4.4. **Financial and economy studies.** The economic order quantity (EOQ) model is a relation that minimizes the total sum of all cost functions, which can be calculated by $Q = \sqrt{2AB/C}$, where A is the annual demand quantity, B is the fixed cost per item and C is the annual holding cost per item. In 2018, Ibrahim and Hadid [30] introduced the ODE model of EOQ

$$\begin{aligned} \frac{d}{d\tau}Q(\tau) &= \frac{Q(\tau)}{2}\left(\frac{A'(\tau)}{A(\tau)} + \frac{B'(\tau)}{B(\tau)} - \frac{C'(\tau)}{C(\tau)}\right) \\ &:= Q(\tau)\left(f_1(\tau) + f_2(\tau) + f_3(\tau)\right) \\ &:= Q(\tau)F(\tau, A, B, C). \end{aligned} \tag{11}$$

Then followed by a set of generalizations by using fractional calculus [31], conformable calculus [32] and fractional differential-difference Dunkl operator [33]. The minimization technique is recognized by using the fixed point theory and its numerical approaches

$$\Delta^{\alpha}Q(\tau) = Q(\tau)F(\tau, A, B, C). \tag{12}$$

4.5. **Epidemiology (COVID-19).** Discrete dynamic systems of SISE were extensively discussed for a long historical period, that successfully described the procedure in disease diffusion. Mainly, by utilizing the system: $d\chi(t)/dt = \chi(t)$, where

χ represents the number of infected people, the rampant phase, the increasing number of asymptomatic infected persons are described. This system is generalized as follows:

(I) By using the conformable calculus [34]

$$\begin{aligned} \mathcal{D}^{\nu}\chi(t) &= \sigma_1(\nu,t)\chi(t) + \sigma_0(\nu,t)\chi'(t) + \sigma(t)\Lambda(t) \\ \mathcal{D}^{\mu}\Lambda(t) &= \rho_1(\mu,t)\Lambda(t) + \rho_0(\mu,t)\Lambda'(t) + \rho(t)\chi(t), \end{aligned} \tag{13}$$

where σ and ρ are the connection rate functions of Λ in $\mathcal{D}^{\nu}\chi(t)$ and χ in $\mathcal{D}^{\mu}\Lambda(t)$ respectively. They describe the damping properties in line for the control energy.

(II) By using a fractional calculus [35]

$$\begin{aligned} \Delta^{\nu}\varphi(t) &= \alpha_1(t)\varphi(t) + \alpha(t)\psi(t) \\ \Delta^{\nu}\psi(t) &= \beta_1(t)\psi(t) + \beta(t)\varphi(t), \end{aligned} \tag{14}$$

where α, α_1 and β, β_1 are the connection rate continuous functions of ψ in $\Delta^{\mu}\varphi(t)$ and φ in $\Delta^{\nu}\psi(t)$ respectively.

(III) By using a fractional entropy (Tsallis entropy)

$$\begin{aligned} S_\jmath(t+1) &= S_\jmath(t)\Big(1 - \tau\jmath\mathcal{T}\ \Upsilon_\alpha(I_\jmath(t))\Big) + \varrho\mathcal{T}\, I_\jmath(t) \\ I_\jmath(t+1) &= I_\jmath(t)\Big(1 - \varrho\mathcal{T}\ \Big) + \tau\jmath\mathcal{T}\, S_\jmath(t)\, \Upsilon_\alpha(I_\jmath(t)), \\ &\Big(0 \le S_\jmath(0) \le N_\jmath,\ \ 0 \le I_\jmath(0) \le N_\jmath\Big) \end{aligned} \tag{15}$$

where $\Upsilon_\alpha(I_\jmath(t))$ is the Tsallis entropy introduced by the probability that any given connect points to an infected node and $\mathcal{T}$ indicates the time-step measure.

4.6. **Computer science.** In image processing, the fractional calculus and classes of fractional differential equations and fractional partial differential equations have been used widely (see [37]-[42]). They were indicated the improvement of many factors in the image, such as the edge detection, denoising, filtering (fractional filter), creative fractional windows, and enhancement of the image.

References

[1] G. S. Salagean, "Subclasses of univalent functions." In Complex Analysis-Fifth Romanian-Finnish Seminar, 362-372, 1983.

[2] E. Sumer, and S. Owa. "Certain classes of analytic functions involving Salagean operator." J. Inequal. Pure Appl. Math Vol. 10, 12-22, 2009.

[3] H. M. Srivastava, S. Owa. Univalent functions, fractional calculus, and their applications. Ellis Horwood; New York; Toronto: Halsted Press, 1989.

[4] R. W. Ibrahim, M. Darus. "Subordination and superordination for univalent solutions for fractional differential equations." Journal of Mathematical Analysis and Applications Vol.345, 871-879, 2008.

[5] R. W. Ibrahim, "Existence and uniqueness of holomorphic solutions for fractional Cauchy problem." Journal of Mathematical Analysis and Applications Vol. 380, 232-240, 2011.

[6] R. W. Ibrahim, "On generalized Srivastava-Owa fractional operators in the unit disk." Advances in Difference Equations Vol. 2011, 1–10, 2011.

[7] R. Tremblay, "Une Contribution a la Theorie de la Derivee Fractionnaire." PhD-T (1974).

[8] R. W. Ibrahim, and J. M. Jahangiri. "Boundary fractional differential equation in a complex domain." Boundary Value Problems Vol. 2014, 1-11, 2014.

[9] D. R. Anderson, and D. J. Ulness. "Newly defined conformable derivatives." Adv. Dyn. Syst. Appl Vol. 10, 109-137, 2015.

[10] R. W. Ibrahim, J. M. Jahangiri. "Conformable differential operator generalizes the Briot-Bouquet differential equation in a complex domain." AIMS Math Vol.4, 1582-1595, 2019.
[11] R. W. Ibrahim, R. M. Elobaid, and S. J. Obaiys. "A Class of Quantum Briot-Bouquet Differential Equations with Complex Coefficients." Mathematics Vol. 8, 1–10, 2020.
[12] R. w. Ibrahim, S. B. Hadid, and S. Momani. "Generalized Briot–Bouquet differential equation by a quantum difference operator in a complex domain." International Journal of Dynamics and Control Vol. 2020, 1-10, 2020.
[13] R. W. Ibrahim, and D. Baleanu. "Entire solutions of a class of algebraic Briot-Bouquet differential equations utilizing majority concept." Advances in Difference Equations Vol. 2020, 1-12, 2020.
[14] R. W. Ibrahim, "Geometric process solving a class of analytic functions using q-convolution differential operator." Journal of Taibah University for Science Vol. 14, 670-677, 2020.
[15] R. W. Ibrahim, Rabha W., and D. Baleanu. "On quantum hybrid fractional conformable differential and integral operators in a complex domain." Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas Vol.115,1-13,2021.
[16] R.W. Ibrahim, and Y. K. Salih. "On a fractional multi-agent cloud computing system based on the criteria of the existence of fractional differential equation." Mathematical Sciences Vol.11, 211-217, 2017.
[17] R.W. Ibrahim, "Maximize minimum utility function of fractional cloud computing system based on search algorithm utilizing the Mittag-Leffler sum." International Journal of Analysis and Applications Vol. 16, n 125-136, 2018.
[18] R.W. Ibrahim, "Utility function for intelligent access web selection using the normalized fuzzy fractional entropy." Soft Computing Vol. 2020, 1-8, 2020.
[19] R. W. Ibrahim, "Time-Delay Multi-Agent Systems for a Movable Cloud." In Computational Mathematics and Variational Analysis, Vol. 2020, 343-356, 2020.
[20] M. Chandrashekhar, et al., An Efficient Provably Secure Verifier-based Authentication Protocol using Fractional Chaotic Maps in Telecare Medicine Information Systems. Soft Computing, 2021.
[21] R. Olfati-Saber, A. Fax, and R. M. Murray. "Consensus and cooperation in networked multi-agent systems." Proceedings of the IEEE Vol. 95, 215-233, 2007.
[22] R. W. Ibrahim, and A. Ghani. "Hybrid cloud entropy systems based on Wiener process." Kybernetes (2016).
[23] R. W. Ibrahim, and A. Gani. "Stability of an iterative fractional multi-agent system." Neural Computing and Applications Vol. 31, 1233-1238, 2019.
[24] M.Chandrashekhar, R. W. Ibrahim, A. J. Obaid, S. G. Meshram, A. Meshram, and A. Mohamed Abd El-Latif. "Fractional chaotic maps based short signature scheme under human-centered IoT environments." Journal of Advanced Research (2020).
[25] C. Tan C, S.H, Niemi L.A. You, Synthetic biology challenge: making cells compute. Molecular BioSystems. Vol. 3(5), 343-53, 2007.
[26] R. W. Ibrahim, and D. Altulea. "Controlled homeodynamic concept using a conformable calculus in artificial biological systems." Chaos, Solitons & Fractals Vol.140 1–15, 2020.
[27] R. W.WIbrahim, "Conformal geometry of the turtle shell." J. King Saud Univ. Sci Vol. 32 2202-2206, 2020.
[28] G. Domokos, G., and P.L. Varkonyi, Geometry and self-righting of turtles, Proceedings of the Royal Society B: Biological Sciences Vol. 275.1630, 11-17, 2007.
[29] R. W. Ibrahim, "Arched foot based on conformal complex neural network testing." Mathematics and Computers in Simulation, Vol. 174, 175-182, 2020 .
[30] R. W. Ibrahim, S. Hadid. "The EOQ model: a differential cyclic system for calculating economic order quantity." International Journal of Analysis and Applications Vol.16, 437-444, 2018.
[31] R. W. Ibrahim, H. Jafari, H. A. Jalab, and S. B. Hadid. "Local fractional system for economic order quantity using entropy solution." Advances in Difference Equations Vol. 2019, 1–10, 2019.
[32] S. B. Hadid, R. W. Ibrahim, and S. Momani. "On The Entropic Order Quantity Model based on The Conformable Calculus." Progr. Fract. Differ. Appl. Vol.7, 1-11, 2021.
[33] R. W. Ibrahim, R. M. Elobaid, and S. J. Obaiys. "A new model of economic order quantity associated with a generalized conformable differential-difference operator." Advances in Difference Equations Vol. 2020, no. 1 (2020): 1-11, 2020.

[34] R. W. Ibrahim, D. Altulea, and R. M. Elobaid. "Dynamical system of the growth of COVID-19 with controller." Advances in Difference Equations Vol. 2021, 1–12, 2021.

[35] S. B. Hadid, R. W. Ibrahim, D. Altulea, and S. Momani. "Solvability and stability of a fractional dynamical system of the growth of COVID-19 with approximate solution by fractional Chebyshev polynomials." Advances in Difference Equations Vol. 1, 1–16, 2020.

[36] M. Shaher, R. W. Ibrahim, and S. B. Hadid. "Susceptible-infected-susceptible epidemic discrete dynamic system based on Tsallis entropy." Entropy Vol. 22, 1–9, 2020.

[37] A. Al-Shamasneh, H. A. Jalab, P. Shivakumara, R. W. Ibrahim, and U. H. Obaidellah. "Kidney segmentation in MR images using active contour model driven by fractional-based energy minimization." Signal, Image and Video Processing 1-8, 2020.

[38] A.H.Altulea, H. A. Jalab, and R. W. Ibrahim. "Fractional Holder mean-based image segmentation for mouse behavior analysis in conditional place preference test." Signal, Image and Video Processing Vol.14, 135-142, 2020.

[39] A. M. Hasan, M. M. AL-Jawad, H. A. Jalab, H. Shaiba, R. W. Ibrahim, and A.A. R. AL-Shamasneh. "Classification of Covid-19 Coronavirus, Pneumonia and Healthy Lungs in CT Scans Using Q-Deformed Entropy and Deep Learning Features." Entropy Vol.22, 1–13, 2020.

[40] R. W. Ibrahim, "A new image denoising model utilizing the conformable fractional calculus for multiplicative noise." SN Applied Sciences Vol. 2, 1–10, 2020.

[41] C. Nath, P. Shivakumara, H. A. Jalab, R. W. Ibrahim, U. Pal, and T. Lu. "A new Fractal Series Expansion based enhancement model for license plate recognition." Signal Processing: Image Communication Vol.89, 1-11, 2020.

[42] A. Hasan, H. A. Jalab, R. W. Ibrahim, F. Meziane, A. R. AL-Shamasneh, and S. J. Obaiys. "MRI brain classification using the quantum entropy LBP and deep-learning-based features." Entropy Vol. 22, 1- 9, 2020.

Rabha W. Ibrahim

IEEE: 94086547, Kuala Lumpur, 59200, Malaysia

E-mail address: `rabhaibrahim@yahoo.com`