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CERTAIN INTEGRAL FORMULAE INVOLVING GENERALIZED HURWITZ-LERCH ZETA FUNCTIONS

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Abstract. In this paper we have established three integral formulae involving λ-generalized Hurwitz-Lerch zeta function. The results are established with the help of generalized Leibnitz rules for integration of fractional order and using the Riemann-Liouville fractional derivatives. Certain new results involving simpler forms of Hurwitz-Lerch zeta function are also obtained as the special cases of our main results.

1. Introduction and Preliminaries

The Riemann-Liouville (Fractional differintegral) operator D_z^{μ} defined by [5, 12]

$$\begin{aligned} D_z^{\mu}\left\{f(z)\right\} &= \frac{1}{\Gamma(-\mu)}\int_0^z (z-\varsigma)^{-\mu-1} f(\varsigma)d\varsigma, \quad (\Re(\mu)<0) \\ &= \frac{d^m}{dz^m} D_z^{\mu-m}\left\{f(z)\right\}, \quad (m-1 \le \Re(\mu) < m; \in \mathbb{N}) \end{aligned} \tag{1}$$

provided that the integral in (1) exists and $\mathbb{N}$ being the set of positive integers.

The extension of classical Leibniz rule for fractional calculus is

$$D_z^{\mu}\{f(z)g(z)\} = \sum_{n=0}^{\infty} \binom{\mu}{n} D_z^{\mu-n}\left\{f(z)\right\} D_z^{\mu}\{g(z)\}, \qquad (\mu \in C). \tag{2}$$

The extended Leibniz rule (2) has a drawback that, interchange of the functions $f(z)$ and $g(z)$ on the right-hand side is not obvious. Therefore, a further generalization of (2) is given by the following bilateral series,

$$D_z^{\mu}\left\{f(z)g(z)\right\} = \sum_{n=-\infty}^{\infty} \binom{\mu}{\upsilon+n} D_z^{\mu-\upsilon-n}\{f(z)\} D_z^{\upsilon+n}\{g(z)\}, \quad (\mu,\upsilon \in C). \tag{3}$$

The above result (3) is the special case of the following generalized Leibniz rule [5][p.317]:

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$$D_z^{\mu}\{f(z)g(z)\} = \sum_{n=-\infty}^{\infty} k \begin{pmatrix} \mu \\ \upsilon + kn \end{pmatrix} D_z^{\mu-\upsilon-kn}\{f(z)\} D_z^{\upsilon+kn}\{g(z)\}, \quad (\mu, \upsilon \in C; 0 < k \leq 1) \tag{4}$$

Where

$$\begin{pmatrix} \mu \\ \upsilon \end{pmatrix} = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\upsilon+1)\Gamma(\upsilon+1)} = \begin{pmatrix} \mu \\ \mu-\upsilon \end{pmatrix}, \qquad (\mu, \upsilon \in C). \tag{5}$$

An integral analogue of the Leibniz rule (2) is also known ([11], see also [3])

$$D_z^{\mu}\{f(z)g(z)\} = \int_{-\infty}^{\infty} \begin{pmatrix} \mu \\ \omega \end{pmatrix} D_z^{\mu-\omega}\{f(z)\} D_z^{\omega}\{g(z)\} d\omega, \qquad (\mu \in C). \tag{6}$$

In which interchange of the function $f(z)$ and $g(z)$ is obviously permissible on the right-hand side as well.

When the integration variable ω is replaced by $\omega + \upsilon$ $(\upsilon \in C)$ or more generally by $k\omega + \upsilon$ $(\upsilon \in C; k > 0)$, (6) readily assume its equivalent forms:

$$D_z^{\mu}\{f(z)g(z)\} = \int_{-\infty}^{\infty} \begin{pmatrix} \mu \\ \upsilon + \omega \end{pmatrix} D_z^{\mu-\upsilon-\omega}\{f(z)\} D_z^{\upsilon+\omega}\{g(z)\} d\omega, \quad (\mu, \upsilon \in C). \tag{7}$$

and

$$D_z^{\mu}\{f(z)g(z)\} = \int_{-\infty}^{\infty} k \begin{pmatrix} \mu \\ \upsilon + k\omega \end{pmatrix} D_z^{\mu-\upsilon-k\omega}\{f(z)\} D_z^{\upsilon+k\omega}\{g(z)\} d\omega, \quad (\mu, \upsilon \in \mathbb{C}). \tag{8}$$

The fractional derivative of power function is an elementary result available in the literature [[6], p.195, Theorem 2]

$$D_z^{\mu}(z^{\lambda}) = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\mu+1)} z^{\lambda-\mu}, \quad (\mu \in C; \lambda \in C/z^{-}; z^{-} := \{-1, -2, -3, ...\}). \tag{9}$$

2. The family of Hurwitz-Lerch zeta functions

The Hurwitz-Lerch zeta function is defined [8],

$$\phi(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{(n+a)^s}, \quad a \in C/z_0 \ ; s \in \mathbb{C}, \text{when } |\text{z}| < 1, \ \text{R(s)} > 1. \tag{10}$$

A generalization of the above Hurwitz Lerch zeta function studied in the literature is as follows [2],

$$\phi_{\mu}^{*}(z, s; a) = \sum_{n=0}^{\infty} \frac{(\mu)_n}{n!} \frac{z^n}{(n+a)^s}, \qquad Re(\mu) > 0 \tag{11}$$

where $(\lambda)_\upsilon$ is a Pochhammer symbol with relation

$$(\lambda)_\upsilon = \frac{\Gamma(\lambda+\upsilon)}{\Gamma(\lambda)}.$$

Further generalization of the Hurwitz-Lerch zeta functions $\phi(z,s,a)$ and $\phi^*_\mu(z,s;a)$ is recently studied in the following form by Garg et al. [1],

$$\phi_{\lambda,\mu,\gamma}(z,s,a) = \sum_{n=0}^{\infty} \frac{(\lambda)_n(\mu)_n}{(\gamma)_n n!}\frac{z^n}{(n+a)^s}. \tag{12}$$

Lin and Srivastava [4] also extended the Hurwitz-Lerch zeta function in the following form,

$$\phi^{\rho,\sigma}_{\mu,\lambda}(z,s,a) = \sum_{n=0}^{\infty} \frac{(\mu)_{\rho n}}{(\lambda)_{\sigma n}}\frac{z^n}{(n+a)^s}. \tag{13}$$

Recently a multiparameter extension known as multiparameter Hurwitz-Lerch zeta function studied by Srivastava [8] is as follows:

$$\begin{aligned}
&\phi^{(\rho_p);(\sigma_q)}_{(\lambda_p);(\mu_q)}(z,s,a) = \phi^{(\rho_1,\ldots,\rho_p;\sigma_1,\ldots,\sigma_q)}_{\lambda_1,\ldots,\lambda_p;\mu_1,\ldots,\mu_q}(z,s,a) = \sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_j)_{\rho_j n}}{\prod_{j=1}^{q}(\mu_j)_{\sigma_j n}}\frac{z^n}{n!(n+a)^s},\\
&p,q\in N_0;\ \lambda_j\in\mathbb{C}\ (j=1,\ldots,p);\ \mu_j\in C/Z_0\ (j=1,\ldots,q)\\
&\rho_j,\sigma_k\in R^+\ (j=1,\ldots,p;k=1,\ldots,q);\ \text{and}\ (\rho_\mathrm{p})\equiv\rho_1,\ldots,\rho_\mathrm{p};(\sigma_\mathrm{q})\equiv\sigma_1,\ldots,\sigma_\mathrm{q}
\end{aligned} \tag{14}$$

and similar representations for $(\lambda_p)\equiv\lambda_1,\ldots,\lambda_p$; $(\mu_q)\equiv\mu_1,\ldots,\mu_q$.

The multiparameter function defined in (14) is generalized and introduced in the following manner by Srivastava and Gaboury [[9], p.1489, equation (10)]

$$\begin{aligned}
&\phi^{(\rho_p);(\sigma_q)}_{(\lambda_p);(\mu_q)}(z,s,a;b,\lambda) = \phi^{(\rho_1,\ldots\rho_p;\sigma_1,\ldots,\sigma_q)}_{\lambda_1,\ldots\lambda_p;\mu_1,\ldots\mu_q}(z,s,a;b,\lambda)\\
&= \frac{1}{\lambda\Gamma s}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_j)_{\rho_j n}}{(a+n)^s\prod\limits_{j=1}^{q}(\mu_j)_{\sigma_j n}} H^{2,\ 0}_{0,\ 2}\left[(a+n)b^{\frac{1}{\lambda}}\Big|\begin{array}{c}---\\(s,1),(0,\frac{1}{\lambda})\end{array}\right]\frac{z^n}{n!}\\
&\min\{Re(a),Re(s)\}>0;\ Re(b)>0;\ \lambda>0;\ \text{where}\ \lambda_\mathrm{j}\in\mathrm{C}(\mathrm{j}=1,\ldots,\mathrm{p})\\
&\text{and}\ \mu_\mathrm{j}\in\mathrm{C}/\mathrm{Z}_0^-(\mathrm{j}=1,\ldots,\mathrm{q});\ \ \rho_\mathrm{j}>0\ \ (\mathrm{j}=1,\ldots,\mathrm{p});\ \sigma_\mathrm{j}>0\ \ (\mathrm{j}=1,\ldots,\mathrm{q});
\end{aligned} \tag{15}$$

and $H(.)$ is the popular Fox's H function.

The generalized function defined in (15) is also introduced and studied earlier by Gupta [[11],p.168,eq.(3.7.16)] in the following form:

$$\phi^{(\alpha,\rho_p,\sigma_q)}_{(\lambda_q,\mu_q,\beta)}(z,s,a,b) = \frac{1}{\Gamma s}\sum_{m=0}^{\infty}\frac{(-b)^m\Gamma(s-\alpha m)}{m!}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\lambda_j)_{\rho_j n}}{\prod_{k=1}^{q}(\mu_k)_{\sigma_k n}}\frac{z^n}{(a+n\beta)^{s-\alpha m}n!}$$
$$p,q>0;\ Re(b)\geq 0;\ \beta,s,z,\lambda_j\in\mathbb{C}\ (j=1,...,p);\ \mu_k\in\mathbb{C}/Z_0^-\ (k=1,...q);$$
$$\rho_j,\sigma_k\in R^+(j=1,...,p;\ k=1,...,q). \tag{16}$$

The integral representation of above generalized Hurwitz-Lerch zeta function is given by Gupta [[11],p.172, eq.(3.8.7)], see also [9]

$$\begin{aligned}&\phi^{(\rho_1,\dots,\rho_p;\sigma_1,\dots,\sigma_q)}_{\lambda_1,\dots,\lambda_p;\mu_1,\dots,\mu_q}(z,s,a;b,\lambda)\\ &= \frac{1}{\Gamma(s)}\int_0^{\infty} t^{s-1}\exp(-at-\frac{b}{t^{\lambda}})_p\Psi_q^*\left[\begin{array}{l}(\lambda_1,\rho_1),...,(\lambda_p,\rho_p);\\(\mu_1,\sigma_1),...,(\mu_q,\sigma_q);\end{array} ze^{-t}\right]dt\\ &\min\{R(a)\ ,R(s)\}\ >0\ ;\ \mathrm{R}\,(b)\geq 0;\ \ \lambda\geq 0\end{aligned} \tag{17}$$

Where

$${}_p\psi_q^*\left[\begin{array}{l}(\lambda_1,\rho_1),...,(\lambda_p,\rho_p);\\(\mu_1,\sigma_1),...,(\mu_q,\sigma_q);\end{array} z\right] = \sum_{n=0}^{\infty}\frac{(\lambda_1)_{\rho_1 n}...(\lambda_p)_{\rho_p n}}{(\mu_1)_{\sigma_1 n}...(\mu_q)_{\sigma_q n}}\frac{z^n}{n!}. \tag{18}$$

The fractional derivative of the Hurwitz-Lerch Zeta function defined in (15) is also required here [10] (see also Gupta [[11], p.175, eqn. (3.8.16)]),

$$\begin{aligned}&D_z^{\alpha}\left\{z^{\beta-1}\phi^{(\rho_1,\dots,\rho_p;\ \sigma_1,\dots,\sigma_q)}_{\lambda_1,\dots,\lambda_p;\ \mu_1,\dots,\mu_q}(z,s,a;b,\lambda)\right\}\\ &= \frac{\Gamma(\beta)}{\Gamma(\beta-\alpha)}z^{\beta+\alpha-1}\phi^{(\rho_1,\dots,\rho_p,1;\ \sigma_1,\dots,\sigma_q,1)}_{\lambda_1,\dots,\lambda_p,\nu;\ \mu_1,\dots,\mu_q,\beta-\alpha}(z,s,a;b,\lambda)\\ &\qquad \lambda>0;\Re(\beta)>0\end{aligned} \tag{19}$$

Certain operators of fractional calculus can be applied with a view to evaluating various families of infinite integrals associated with functions of one and several variables. Motivated by the work of Jaimini et al. [3] on the integral analogue of the Leibnit z rule for fractional calculus, we have established here certain infinite integrals involving recently defined λ-Hurwitz-Lerch zeta function due to Srivastava [9].

Throughout the paper $(\rho_p)\equiv\rho_1,...,\rho_p;\ (\lambda_p)\equiv\lambda_1,...,\lambda_p;\ (\sigma_q)\equiv\sigma_1,...,\sigma_q$and $(\mu_q)\equiv\mu_1,...,\mu_q$.

3. Main Results

The following three integral formulae are established here

$$\begin{aligned}&\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),\upsilon;(\mu_q),\upsilon-\mu}(z,s,a;b,\lambda)\\ &=\Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}\left(\begin{array}{c}\mu\\ \omega\end{array}\right)\frac{1}{\Gamma(1-\omega)\Gamma(\upsilon-\mu+\omega)}\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),1;(\mu_q),1-\omega}(z,s,a;b,\lambda)d\omega\\ &\min\{R(a),R(s)\}>0, Re(b)>0,\ \upsilon>0,\ \lambda>0;\ \mu\in\mathbb{C},\end{aligned} \tag{20}$$

provided the result in (20) exist.

$$\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),\upsilon;(\mu_q),\upsilon-\mu}(z,s,a;b,\lambda)$$
$$= \Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}\left(\begin{array}{c}\mu\\ \tau+\omega\end{array}\right)\frac{1}{\Gamma(1-\tau-\omega)\Gamma(\tau+\upsilon-\mu+\omega)}\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),1;(\mu_q),1-\omega-\tau}(z,s,a;b,\lambda)d\omega$$
$$\min\{R(a),R(s)\}>0, Re(b)>0,\ \upsilon>0,\ \lambda>0;\ \mu,\tau\in\mathbb{C} \tag{21}$$

provided the result in (21) exist.

$$\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),\upsilon;(\mu_q),\upsilon-\mu}(z,s,a;b,\lambda)$$
$$= \Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}k\left(\begin{array}{c}\mu\\ \tau+k\omega\end{array}\right)\frac{1}{\Gamma(1-\tau-k\omega)\Gamma(\tau+\upsilon-\mu+k\omega)} \tag{22}$$
$$\times\,\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),1;(\mu_q),1-\tau-k\omega}(z,s,a;b,\lambda)d\omega$$
$$\min\{R(a),R(s)\}>0, Re(b)>0,\upsilon>0,\lambda>0;\ \mu,\tau\in C;\ 0<k\le 1$$

provided the result in (22) exist.

3.1. **Outline of proofs.** To prove the result (20), we first set in (6)

$$f(z)=z^{\upsilon-1}, \tag{23}$$

$$g(z)=\phi^{(\rho_p);(\sigma_q)}_{(\lambda_p);(\mu_q)}(z,s,a;b,\lambda) \tag{24}$$

we obtain

$$D_z^{\mu}\left\{z^{\upsilon-1}\phi^{(\rho_p);(\sigma_q)}_{(\lambda_p);(\mu_q)}(z,s,a;b,\lambda)\right\}$$
$$=\int_{-\infty}^{\infty}\left(\begin{array}{c}\mu\\ \omega\end{array}\right)D_z^{\mu-\omega}\left\{z^{\upsilon-1}\right\}D_z^{\omega}\left\{\phi^{(\rho_p);(\sigma_q)}_{(\lambda_p);(\mu_q)}(z,s,a;b,\lambda)\right\}d\omega. \tag{25}$$

Now, on evaluating the fractional derivatives appearing on both sides in (25), using (9) and (19) therein, we at once arrive at the desired result in (20).

The results in (21) and (22) are proved, following the similar lines as to prove the result (20) by setting $f(z)$ and $g(z)$ given in (23) and (24) respectively in (7) for (21) and (8) for (22).

4. Special Cases

(1) If in results (20), (21) and (22) we take $b=0$, these results reduce respectively to the following new results involving Hurwitz-Lerch zeta function defined in (14):

$$\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),\upsilon;(\mu_q),\upsilon-\mu}(z,s,a)$$
$$=\Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}\left(\begin{array}{c}\mu\\ \omega\end{array}\right)\frac{1}{\Gamma(1-\omega)\Gamma(\upsilon-\mu+\omega)}\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),1;(\mu_q),1-\omega}(z,s,a)d\omega \tag{26}$$
$$\min\{Re(a),Re(s)\}>0,\ \mu\in\mathbb{C},\ \upsilon>0.$$

$$\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),\upsilon;(\mu_q),\upsilon-\mu}(z,s,a) = \Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}\binom{\mu}{\tau+\omega}\frac{1}{\Gamma(1-\tau-\omega)\Gamma(\tau+\upsilon-\mu+\omega)} \times \phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_P),1;(\mu_q),1-\omega-\tau}(z,s,a)d\omega, \tag{27}$$

$$\min\{Re(a),Re(s)\} > 0, \quad \mu,\tau\in C, \ \ \upsilon>0.$$

$$\phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_p),\upsilon;(\mu_q),\upsilon-\mu}(z,s,a) = \Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}k\binom{\mu}{\tau+k\omega}\frac{1}{\Gamma(1-\tau-k\omega)\Gamma(\tau+\upsilon-\mu+k\omega)} \times \phi^{(\rho_p),1;(\sigma_q),1}_{(\lambda_P),1;(\mu_q),1-k\omega-\tau}(z,s,a)d\omega \tag{28}$$

$$\min\{Re(a),Re(s)\} > 0, \quad \mu,\tau\in C, \ \ \upsilon>0, \ \ 0<k\le 1.$$

(2) If in results (20), (21) and (22), we take $p=\rho_1=\lambda_1=1$ and $q=b=0$ these results reduce respectively to the following new results involving Hurwitz-Lerch zeta function due to Lin and Srivastava [4] defined as in (13):

$$\phi^{1,1}_{\upsilon,\upsilon-\mu}(z,s,a) = \Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}\binom{\mu}{\omega}\frac{1}{\Gamma(1-\omega)\Gamma(\upsilon-\mu+\omega)}\phi^{1,1}_{1,1-\omega}(z,s,a)d\omega \tag{29}$$

$$\min\{Re(a),Re(s)\} > 0, \quad \mu\in C, \ \ \upsilon>0.$$

$$\phi^{1,1}_{\upsilon,\upsilon-\mu}(z,s,a) = \Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}\binom{\mu}{\tau+\omega}\frac{1}{\Gamma(1-\omega-\tau)\Gamma(\upsilon-\mu+\omega+\tau)}\phi^{1,1}_{1,1-\omega-\tau}(z,s,a)d\omega \tag{30}$$

$$\min\{Re(a),Re(s)\} > 0, \quad \mu,\tau\in C, \ \ \upsilon>0.$$

$$\phi^{1,1}_{\upsilon,\upsilon-\mu}(z,s,a) = \Gamma(\upsilon-\mu)\int_{-\infty}^{\infty}k\binom{\mu}{\tau+k\omega}\frac{1}{\Gamma(1-k\omega-\tau)\Gamma(\upsilon-\mu+k\omega+\tau)}\phi^{1,1}_{1,1-k\omega-\tau}(z,s,a)d\omega \tag{31}$$

$$\min\{Re(a),Re(s)\} > 0, \quad \mu,\tau\in C, \ \ \upsilon>0, \ \ 0<k\le 1.$$

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