*Journal of Fractional Calculus and Applications*,
Vol. 10(2) July 2019, pp. 31-39.
ISSN: 2090-5858.
http://math-frac.oreg/Journals/JFCA/

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# ELLIPTIC WELL-POISED BAILEY LEMMA AND ITS APPLICATIONS

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Abstract. In this paper, we have established a theorem by using the elliptic WP-Bailey lemma. Certain transformation formulae for elliptic hypergeometric series have also been obtained by making use of the theorem established herein.

## 1. Introduction

Bailey in 1947 established a remarkable lemma which has been widely used for obtaining transformation formulas for ordinary hypergeometric series as well as for basic hypergeometric series. In order to obtain Rogers-Ramanujan type identities Bailey introduced Bailey pair. Using Bailey lemma and Bailey pairs Andrews [1], Bailey [2, 3], Denis, Remy Y., Singh, S.N. and Singh, S.P. [4, 5, 6], Slater [16, 17], Verma [21] established a number of transformations and identities involving q series. Andrews generalized Bailey pair and introduced WP-Bailey pair, WP-Bailey chain and WP-Bailey tree. Making use of WP-Bailey pairs, several mathematicians attempted to establish new transformations and identities for basic hypergeometric series. Noteworthy works in this direction are due to Laughlin [10, 11], Singh, S.N., Singh, Sunil and Singh, Priyanka [15], Srivastava, H.M., Singh, S.N., Singh, S.P. and Yadav, Vijay [19, 20].

Later on in 2002 Spiridonov [18], Warnaar [23] extended the idea of WP-Bailey pairs and introduced elliptic well-poised Bailey lemma and elliptic WP-Bailey chain. Many useful summations and transformations for elliptic hypergeometric series have been established by Spiridonov [18], Warnaar [22], Frankel, I.B. and Turaev, V.G. [7], Singh, Satya Prakash, Singh, Ashutosh and Singh Dhirendra [12], Singh, Satya Prakash, Mishra, Bindu Prakash, Mohd. Shahjade and Yadav, Vijay [13], Singh, S.N., Singh, Priyanka and Sharma, Mahendra Kumar [14] and others [24-35].

Elliptic hypergeometric series and their extensions to theta hypergeometric series has become an increasingly active area of research now these days. So for, many formulas for very well-poised basic hypergeometric series have already been extended

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2010 *Mathematics Subject Classification.* 33D15, 33D10, 33E05.
*Key words and phrases.* Elliptic hypergeometric series, elliptic WP-Bailey lemma, elliptic WP-Bailey pair, transformation formula, summation formula.
Submitted May 25, 2018.

to the elliptic setting. Some interesting formulas for multi-basic hypergeometric series appear in the work of Warnaar [23], Singh, Srivasatava, et.al.[19, 20].

In the present paper, we establish a theorem which will be used to obtain transformation and summation formulas for elliptic hypergeometric series.

## 2. Notations and Definitions

A modified Jacobi's theta function with argument x and nome p is defined by,

$$\theta(x;p) = (x, p/x; p)_\infty = (x;p)_\infty (p/x;p)_\infty, \tag{1}$$

and

$$(x;p)_\infty = \prod_{r=0}^{\infty}(1 - xp^r).$$

Also,

$$\theta(x_1, x_2, ..., x_r; p) = \theta(x_1;p)\theta(x_2;p)...\theta(x_r;p) \tag{2}$$

Following Gasper and Rahman [[8]; chapter 11] theta shifted factorial is defined by,

$$(a;q,p)_n = \theta(a;p)\theta(aq;p)...\theta(aq^{n-1};p)$$

,

$$(a;q,p)_0 = 1$$

and

$$(a;q,p)_{-n} = \frac{(-1)^n q^{n(n+1)/2}}{a^n (q/a;q,p)_n}. \tag{3}$$

For the sake of brevity, we often write,

$$(a_1, a_2, ..., a_r; q, p)_n = (a_1;q,p)_n (a_2;q,p)_n ... (a_r;q,p)_n,$$

where $a_1, a_2, ..., a_r \neq 0$.

Following Spiridonov [18], we define an ${}_{r+1}E_r$ theta hypergeometric series with base q and nome p by,

$${}_{r+1}E_r\left[\begin{matrix} a_1, a_2, ..., a_{r+1}; q, p; z \\ b_1, b_2, ..., b_r \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, ..., a_{r+1}; q, p)_n z^n}{(q, b_1, b_2, ..., b_r; q, p)_n}, \tag{4}$$

where a's and b's are never zero. If z, a's and b's are independent of p then

$$\lim_{p\to 0}\ {}_{r+1}E_r\left[\begin{matrix} a_1, a_2, ..., a_{r+1}; q, p; z \\ b_1, b_2, ..., b_r \end{matrix}\right] = {}_{r+1}\Phi_r\left[\begin{matrix} a_1, a_2, ..., a_{r+1}; q; z \\ b_1, b_2, ..., b_r \end{matrix}\right]. \tag{5}$$

The theta hypergeometric series ${}_{r+1}E_r$ defined in (4) becomes an elliptic hypergeometric series with two fundamental periods $\sigma^{-1}$ and $\tau/\sigma$ provided

$$a_1 a_2 ... a_{r+1} = q b_1 b_2 ... b_r, \tag{6}$$

where $q = e^{2\pi i\sigma}$ and $p = e^{2\pi i\tau}$.

The elliptic hypergeometric series ${}_{r+1}E_r$ is called well poised if,

$$qa_1 = b_1 a_2 = b_2 a_3 = ... = b_r a_{r+1}. \tag{7}$$

In this case elliptic balancing condition

$$a_1 a_2 a_3 ... a_{r+1} = q b_1 b_2 ... b_r$$

reduces to

$$(aq)^{r+1} = (a_1 a_2 ... a_{r+1})^2. \tag{8}$$

Following Spiridonov [18], the well-poised theta hypergeometric series is defined by,

$$
{}_{r+1}V_r[a_1; a_6, a_7, ..., a_{r+1}; q, p; z] = \sum_{n=0}^{\infty} \frac{\theta(aq^{2n}; p)(a_1, a_6, a_7, ..., a_{r+1}; q, p)_n (zq)^n}{\theta(a; p)(q, a_1q/a_6, ..., a_1q/a_{r+1}; q, p)_n}
$$

$$
= \sum_{n=0}^{\infty} \frac{(a_1, q\sqrt{a_1}, -q\sqrt{a_1}, q\sqrt{\frac{a_1}{p}}, -q\sqrt{a_1p}, a_6, ..., a_{r+1}; q, p)_n (-z)^n}{(q, \sqrt{a_1}, -\sqrt{a_1}, \sqrt{a_1p}, -\sqrt{\frac{a_1}{p}}, \frac{a_1q}{a_r}, ..., \frac{a_1q}{a_{r+1}}; q, p)_n}. \tag{9}
$$

If the argument z in ${}_{r+1}V_r$ is 1 then we supress 1 and denote it by,

$$
{}_{r+1}V_r[a_1; a_6, a_7, ..., a_{r+1}; q, p]. \tag{10}
$$

We shall make use of the following summation formula in our analysis,

$$
{}_{10}V_9[a; b, c, d, e, q^{-n}; q, p] = \frac{\left(aq, \frac{aq}{bc}, \frac{aq}{bd}, \frac{aq}{cd}; q, p\right)_n}{\left(\frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d}, \frac{aq}{bcd}; q, p\right)_n}, \tag{11}
$$

provided that $bcde = a^2q^{n+1}$.

[Gasper & Rahman 8; (11.2.25) p. 307]

## 3. Elliptic Extension of WP-Bailey Lemma

Following Warnaar [23] elliptic extension of WP-Bailey pair is defined as;

A pair of sequence $\langle \alpha_n(a, k; q, p), \beta_n(a, k; q, p) \rangle$ is said to be elliptic WP-Bailey pair if

$$
\beta_n(a, k; q, p) = \sum_{r=0}^{n} \frac{(k/a; q, p)_{n-r}(k; q, p)_{n+r}}{(q; q, p)_{n-r}(aq; q, p)_{n+r}} \alpha_r(a, k; q, p) \tag{12}
$$

Similarly, a pair of sequences $\langle \gamma_n(a, k; q, p), \delta_n(a, k; q, p) \rangle$ is said to be elliptic conjugate WP-Bailey pair if,

$$
\gamma_n(a, k; q, p) = \sum_{r=0}^{\infty} \frac{(k/a; q, p)_r(k; q, p)_{r+2n}}{(q; q, p)_r(aq; q, p)_{r+n}} \delta_{r+n}(a, k; q, p), \tag{13}
$$

provided the infinite series converges.

Again, following Bailey lemma we have;

If $\langle \alpha_n(a, k; q, p), \beta_n(a, k; q, p) \rangle$ is elliptic WP-Bailey pair and the elliptic conjugate WP-Bailey pair is $\langle \gamma_n(a, k; q, p), \delta_n(a, k; q, p) \rangle$, then under suitable convergence conditions, we have,

$$
\sum_{n=0}^{\infty} \alpha_n(a, k; q, p)\gamma_n(a, k; q, p) = \sum_{n=0}^{\infty} \beta_n(a, k; q, p)\delta_n(a, k; q, p). \tag{14}
$$

**Theorem 1.**

If $\langle \alpha_n(a, k; q, p), \beta_n(a, k; q, p) \rangle$ is a elliptic WP-Bailey pair then

$$
\frac{\left(kq, \frac{kq}{bc}, \frac{aq}{b}, \frac{aq}{c}; q, p\right)_N}{\left(aq, \frac{aq}{bc}, \frac{kq}{b}, \frac{kq}{c}; q, p\right)_N} \sum_{n=0}^{N} \frac{\left(b, c, \frac{ak}{bc}q^{1+N}, q^{-N}; q, p\right)_n \left(\frac{a}{k}\right)^n}{\left(\frac{aq}{b}, \frac{aq}{c}, \frac{bc}{k}q^{-N}, aq^{1+N}; q, p\right)_n} \alpha_n(a, k; q, p)
$$

$$= \sum_{n=0}^{N} \frac{\theta(kq^{2n};p)(b,c,akq^{1+N}/bc,q^{-N};q,p)_n \beta_n(a,k;q,p)}{\theta(k;p)(kq/b,kq/c,bcq^{-N}/a,kq^{1+N};q,p)_n}. \tag{15}$$

**Proof.**

Choosing $\delta_r(a,k;q,p) = \dfrac{\theta(kq^{2r};p)(b,c,\frac{ak}{bc}q^{1+N};q,p)_r(1/k;q,p)_{-N-r}}{\theta(k;p)(\frac{kq}{b},\frac{kq}{c},\frac{bc}{a}q^{-N};q,p)_r(q;q,p)_{N-r}(kq^{2N+1})^r}$ in (13) we get,

$$\gamma_n(a,k;q,p) = \frac{(-k)^N q^{N(N+1)/2}\theta(kq^{2n};p)(k;q,p)_{2n}(b,c,\frac{ak}{bc}q^{1+N},q^{-N};q,p)_n}{(kq,q;q,p)_N\theta(k;p)(aq;q,p)_{2n}(\frac{kq}{b},\frac{kq}{c},\frac{bc}{a}q^{-N},kq^{N+1};q,p)_n} \times$$

$$\times \ {}_{10}V_9\left[kq^{2n};bq^n,cq^n,\frac{ak}{bc}q^{1+n+N},\frac{k}{a},q^{-(N-n)};q,p\right]. \tag{16}$$

Now, summing ${}_{10}V_9$ series by using (11) we find,

$$\gamma_n(a,k;q,p) = \frac{(kq/bc,aq/b,aq/c;q,p)_N(-k)^N q^{N(N+1)/2}}{(q,kq/b,kq/c,aq,aq/bc;q,p)_N} \times$$

$$\times \frac{(b,c,akq^{1+N}/bc,q^{-N};q,p)_n}{(aq/b,aq/c,bcq^{-N}/k,aq^{1+N};q,p)_n}\left(\frac{a}{k}\right)^n. \tag{17}$$

Putting the values of $\delta_n(a,k;q,p)$ and $\gamma_n(a,k;q,p)$ in (14) we get (15).

**Applications of (15)**

If we make use of elliptic WP-Bailey pair due to Warnaar [23; (4.2a), (4.2b)] we have the following summation formula,

$$\sum_{n=0}^{\infty} \frac{\theta(aq^{2n};p)}{\theta(a;p)} \frac{\left(a,\frac{a}{k},b,c,\frac{ak}{bc}q^{N+1},q^{-N};q,p\right)_n}{\left(q,kq,\frac{aq}{b},\frac{aq}{c},\frac{bc}{k}q^{-N},aq^{N+1};q,p\right)_n}$$

$$= {}_{10}V_9\left[a;\frac{a}{k},b,c,\frac{ak}{bc}q^{1+N},q^{-N};q,p\right]$$

$$= \frac{\left(aq,\frac{aq}{bc},\frac{kq}{b},\frac{kq}{c};q,p\right)_N}{\left(kq,\frac{kq}{bc},\frac{aq}{b},\frac{aq}{c};q,p\right)_N}. \tag{18}$$

Again, replacing a,q,p by $a^2,q^2,p^2$ respectively in (15) it takes the form,

$$\frac{\left(kq^2,\frac{kq^2}{bc},\frac{a^2q^2}{b},\frac{a^2q^2}{c};q^2,p^2\right)_N}{\left(a^2q^2,\frac{a^2q^2}{bc},\frac{kq^2}{b},\frac{kq^2}{c};q^2,p^2\right)_N} \times$$

$$\times \sum_{n=0}^{N} \frac{\left(b,c,\frac{a^2k}{bc}q^{2+2N},q^{-2N};q^2,p^2\right)_n \left(\frac{a^2}{k}\right)^n \alpha_n(a^2,k;q^2,p^2)}{\left(\frac{a^2q^2}{b},\frac{a^2q^2}{c},\frac{bc}{k}q^{-2N},a^2q^{2+2N};q^2,p^2\right)_n}$$

$$= \sum_{n=0}^{N} \frac{\theta(kq^{4n};p^2)\left(b, c, \frac{a^2k}{bc}q^{2+2N}, q^{-2N}; q^2, p^2\right)_n \beta_n(a^2, k; q^2, p^2)}{\theta(k;p^2)\left(\frac{kq^2}{b}, \frac{kq^2}{c}, \frac{bc}{a^2}q^{-2N}, kq^{2+2N}; q^2, p^2\right)_n}. \tag{19}$$

Using the elliptic WP-Bailey pair due to Warnaar [23], viz.,

$$\alpha_n(a^2, k; q^2, p^2) = \frac{\theta(aq^{2n};p)(a, a^2q/k; q, p)_n}{\theta(a;p)(q, k/a; q, p)_n}\left(\frac{k}{a^2q}\right)^n$$

$$\beta_n(a^2, k; q^2, p^2) = \frac{(-k/a; q, p)_{2n}(k, a^2q^2/k; q^2, p^2)_n}{(-aq; q, p)_{2n}(q^2, k^2/a^2; q^2, p^2)_n}\left(\frac{k}{a^2q}\right)^n$$

in (19) we get the transformation formula,

$$\frac{\left(kq^2, \frac{kq^2}{bc}, \frac{a^2q^2}{b}, \frac{a^2q^2}{c}; q^2, p^2\right)_N}{\left(a^2q^2, \frac{a^2q^2}{bc}, \frac{kq^2}{b}, \frac{kq^2}{c}; q^2, p^2\right)_N} \times$$

$$\times \sum_{n=0}^{N} \frac{\left(b, c, \frac{a^2k}{bc}q^{2+2N}, q^{-2N}; q^2, p^2\right)_n \theta(aq^{2n};p)(a, a^2q/k; q, p)_n}{\left(\frac{a^2q^2}{b}, \frac{a^2q^2}{c}, \frac{bc}{k}q^{-2N}, a^2q^{2+2N}; q^2, p^2\right)_n \theta(a;p)(q, k/a; q, p)_n} q^{-n}$$

$$= \sum_{n=0}^{N} \frac{\theta(kq^{4n};p^2)\left(k, \frac{a^2q^2}{k}, b, c, \frac{a^2k}{bc}q^{2+2N}, q^{-2N}; q^2, p^2\right)_n (-k/a; q, p)_{2n}}{\theta(k;p^2)\left(q^2, \frac{k^2}{a^2}, \frac{kq^2}{b}, \frac{kq^2}{c}, \frac{bc}{a^2}q^{-2N}, kq^{2+2N}; q^2, p^2\right)_n (-aq; q, p)_{2n}}\left(\frac{k}{a^2q}\right)^n. \tag{20}$$

Making use of another elliptic WP-Bailey pair due to Warnaar [23], viz.,

$$\alpha_{2n}(a, k; q, p) = \frac{\theta(aq^{4n};p)(a, a^2/k^2; q^2, p)_n}{\theta(a;p)(q^2, k^2q^2/a; q^2, p)_n}\left(\frac{k^2}{a^2}\right)^n,$$

$$\alpha_{2n+1}(a, k; q, p) = 0$$

$$\beta_n(a, k; q, p) = \frac{(kq^2/a; q^2, p)_n(k, a/k; q, p)_n}{(aq; q^2, p)_n(q, k^2q/a; q, p)_n}\left(-\frac{k}{a}\right)^n$$

in (15) we get

$$\frac{\left(kq, \frac{kq}{bc}, \frac{aq}{b}, \frac{aq}{c}; q, p\right)_N}{\left(aq, \frac{aq}{bc}, \frac{kq}{b}, \frac{kq}{c}; q, p\right)_N} \times$$

$$\times \sum_{n=0}^{N} \frac{\left(b, c, \frac{ak}{bc}q^{1+N}, q^{-N}; q, p\right)_{2n} \theta(aq^{4n};p)(a, a^2/k^2; q^2, p)_n}{\left(\frac{aq}{b}, \frac{aq}{c}, \frac{bc}{k}q^{-N}, aq^{1+N}; q, p\right)_{2n} \theta(a;p)(q^2, k^2q^2/a; q^2, p)_n}$$

$$= \sum_{n=0}^{N} \frac{\theta(kq^{2n};p)\left(b, c, \frac{akq^{1+N}}{bc}, q^{-N}; q, p\right)_n \left(\frac{k^2q}{a}; q^2, p\right)_n \left(k, \frac{a}{k}; q, p\right)_n}{\theta(k;p)\left(\frac{kq}{b}, \frac{kq}{c}, \frac{bcq^{-N}}{a}, kq^{1+N}; q, p\right)_n (aq; q^2, p)_n \left(q, \frac{k^2q}{a}; q, p\right)_n} \left(-\frac{k}{a}\right)^n. \tag{21}$$

Definition of the elliptic WP-Bailey pair (12) can be written as,

$$\beta_n(a, k; q, p) = \frac{(k/a; q, p)_n (k; q, p)_n}{(q; q, p)_n (aq; q, p)_n} \times$$
$$\times \sum_{r=0}^{n} \frac{(q^{-n}; q, p)_r (kq^n; q, p)_r}{(aq^{1-n}/k; q, p)_r (aq^{1+n}; q, p)_r} \left(\frac{aq}{k}\right) \alpha_r(a, k; q, p). \tag{22}$$

Substituting the elliptic WP-Bailey pair due to Warnaar [23], viz.,

$$\alpha_{2n}(a, k; q, p) = \frac{\theta(aq^{4n};p)(a, a^2/m^2; q^2, p)_n (b, c; q, p)_{2n}}{\theta(a;p)(q^2, m^2q^2/a; q^2, p)_n (aq/b, aq/c; q, p)_{2n}} \left(\frac{k}{a}\right)^{2n},$$
$$\alpha_{2n+1} = 0,$$
$$\beta_n(a, k; q, p) = \frac{(k, k/m, bk/a, ck/a; q, p)_n}{(q, mq, aq/b, aq/c; q, p)_n} \sum_{r=0}^{n} \frac{\theta(mq^{2r};p)(m^2q/a; q^2, p)_r}{\theta(m;p)(aq; q^2, p)_r} \times$$
$$\times \frac{(m, b, c, a/m, kq^n, q^{-n}; q, p)_r (-mq/a)^r}{(q, mq/b, mq/c, m^2q/a, mq^{1-n}/k, mq^{1+n}; q, p)_r}.$$

in (22) we get the following transformation formula,

$$\frac{(k/m, bk/a, ck/a; q, p)_n}{(mq, aq/b, aq/c; q, p)_n} \sum_{r=0}^{n} \frac{\theta(mq^{2r};p)(m^2q/a; q^2, p)_r}{\theta(m;p)(aq; q^2, p)_r} \times$$
$$\times \frac{(m, b, c, a/m, kq^n, q^{-n}; q, p)_r (-mq/a)^r}{(q, mq/b, mq/c, m^2q/a, mq^{1-n}/k, mq^{1+n}; q, p)_r}$$
$$= \frac{(k/a; q, p)_n}{(aq; q, p)_n} \sum_{r=0}^{n} \frac{(q^{-n}; q, p)_{2r} (kq^n; q, p)_{2r}}{(aq^{1-n}/k; q, p)_{2r} (aq^{1+n}; q, p)_{2r}} \times$$
$$\times \frac{\theta(aq^{4r};p)(a, a^2/q^2; q^2, p)_r (b, c; q, p)_{2r} q^{2r}}{\theta(a;p)(q^2, m^2q^2/a; q^2, p)_r (aq/b, aq/c; q, p)_{2r}}, \tag{23}$$

where $m = bck/aq$.

## 4. New Elliptic WP-Bailey Pairs

Warnaar [23] has given five theorems for constructing new WP-elliptic pair from a known pair. We shall discuss one of these theorems here.

**Theorem 2 of Warnaar** [23] **States that:**

If $\langle \alpha_n(a, k; q, p), \beta_n(a, k; q, p) \rangle$ is an elliptic WP-Bailey pair then so is the pair $\langle \alpha'_n(a, k; q, p), \beta'_n(a, k; q, p) \rangle$ given by,

$$\alpha'_n(a, k; q, p) = \frac{(b, c; q, p)_n}{(aq/b, aq/c; q, p)_n} \left(\frac{k}{m}\right)^n \alpha_n(a, m; q, p),$$
$$\beta'_n(a, k; q, p) = \frac{(mq/b, mq/c; q, p)_n}{(aq/b, aq/c; q, p)_n} \sum_{r=0}^{n} \frac{\theta(mq^{2r};p)(b, c; q, p)_r}{\theta(m;p)(mq/b, mq/c; q, p)_r} \times$$
$$\times \frac{(k/m; q, p)_{n-r} (k; q, p)_{n+r}}{(q; q, p)_{n-r} (mq; q, p)_{n+r}} \left(\frac{k}{m}\right)^r \beta_n(a, m; q, p), \tag{24}$$

where $m = bck/aq$.

An elliptic WP-Bailey pair due to Warnaar [23] is

$$\alpha_n(a,k;q,p) = \frac{\theta(aq^{2n};p)(a,a/k;q,p)_n}{\theta(a;p)(q,kq;q,p)_n}\left(\frac{k}{a}\right)^n$$

$$\beta_n(a,k;q,p) = \delta_{n,0}. \tag{25}$$

Using (25) in (24) we get new elliptic WP-Bailey pair as,

$$\alpha'_n(a,k;q,p) = \frac{(b,c;q,p)_n}{(aq/b,aq/c;q,p)_n}\left(\frac{k}{m}\right)^n \frac{\theta(aq^{2n};p)(a,a/m;q,p)_n}{\theta(a;p)(q,mq;q,p)_n}\left(\frac{m}{a}\right)^n$$

$$\beta'_n(a,k;q,p) = \frac{(mq/b,mq/c;q,p)_n}{(aq/b,aq/c;q,p)_n}. \tag{26}$$

Putting these values of new elliptic WP-Bailey pair in (22) we have, following summation formula

$${}_{10}V_9[a;b,c,a/m,kq^n,q^{-n};q,p] = \frac{(q,aq,mq/b,mq/c;q,p)_n}{(k,k/a,aq/b,aq/c;q,p)_n}, \tag{27}$$

where $m = bck/aq$.

One can use (25), (26) to establish results as shown in (27).

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