Journal of Fractional Calculus and Applications
Vol. 9(1) Jan. 2018, pp. 167-179.
ISSN: 2090-5858.
http://fcag-egypt.com/Journals/JFCA/

SOME APPLICATION OF A POISSON DISTRIBUTION SERIES ON SUBCLASSES OF UNIVALENT FUNCTIONS

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Abstract. In this paper, we introduce a power series with coefficients are the probabilities of Poisson distribution and obtain sufficient conditions for this power series and some related series to be in various subclasses of analytic functions. Also, we investigate several mapping properties involving these subclasses.

1. Introduction

Let $\mathcal{A}$ denote the class of functions $f(z)$ of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$, and let $\mathcal{S}$ be the subclass of all functions in $\mathcal{A}$, which are univalent. For $g(z) \in \mathcal{A}$ of the form

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \tag{2}$$

the Hadamard product (or convolution) of two power series $f(z)$ and $g(z)$ is given by (see [1])

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z).$$

and the integral convolution is defined by (see [1]):

$$(f \circledast g)(z) = z + \sum_{n=2}^{\infty} \frac{a_n b_n}{n} z^n = (g \circledast f)(z).$$

Definition 1.1. *For two functions $f(z)$ and $g(z)$ analytic in $\mathbb{U}$, we say that the function $f(z)$ is subordinate to $g(z)$ in $\mathbb{U}$ and written $f(z) \prec g(z)$, if there exists a Schwarz function $w(z)$, analytic in $\mathbb{U}$ with $w(0) = 0$ and $w(z) < 1$ such that*

2010 *Mathematics Subject Classification.* for example 30C45, 30C50.

Key words and phrases. Poisson distribution series, Analytic functions, Hadamard product, Starlike function, Integral convolution.

Submitted March 19, 2016.

$f(z) = g(w(z))$ $(z \in \mathbb{U})$. Furthermore, if the function $g(z)$ is univalent in $\mathbb{U}$, then we have the following equivalence (see [7]):

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(\mathbb{U}) \subset g(\mathbb{U}).$$

Let $\mathcal{S}^*(\alpha)$ and $\mathcal{K}(\alpha)$ denote the subclasses of starlike and convex functions of order α, respectively. We note that $\mathcal{S}^*(0) = \mathcal{S}^*$ and $\mathcal{K}(0) = \mathcal{K}$, the subclasses of starlike and convex functions (see [6, 8, 10, 12, 13, 14] and [17]).

Definition 1.2. ([2]) *For $0 \le \alpha < 1$, $\beta \ge 0$, $-1 \le B < A \le 1$, and $g(z)$ is given by (2), we denote $\mathcal{S}(f, g; A, B; \alpha, \beta)$ the subclass of $\mathcal{S}$ consisting of functions of the form (1) and satisfying:*

$$\frac{z(f*g)'(z)}{(f*g)(z)} - \beta\left|\frac{z(f*g)'(z)}{(f*g)(z)} - 1\right| \prec (1-\alpha)\frac{1+Az}{1+Bz} + \alpha.$$

Or equivalently,

$$\left|\frac{\frac{z(f*g)'(z)}{(f*g)(z)} - \beta\left|\frac{z(f*g)'(z)}{(f*g)(z)} - 1\right| - 1}{B\left[\frac{z(f*g)'(z)}{(f*g)(z)} - \beta\left|\frac{z(f*g)'(z)}{(f*g)(z)} - 1\right|\right] - [B + (A-B)(1-\alpha)]}\right| < 1.$$

We note that:

(i): $\mathcal{S}(f, g; A, B; \alpha, 0) = \mathcal{S}(f, g; A, B; \alpha)$

$$= \left\{ f(z) \in \mathcal{S} : \left|\frac{\frac{z(f*g)'(z)}{(f*g)(z)} - 1}{B\frac{z(f*g)'(z)}{(f*g)(z)} - [B + (A-B)(1-\alpha)]}\right| < 1 \ (z \in \mathbb{U}) \right\};$$

(ii): $\mathcal{S}(f, g; \gamma, -\gamma; \alpha, 0) = \mathcal{S}(f, g; \gamma, \alpha)$

$$= \left\{ f(z) \in \mathcal{S} : \left|\frac{\frac{z(f*g)'(z)}{(f*g)(z)} - 1}{\frac{z(f*g)'(z)}{(f*g)(z)} + 1 - 2\alpha}\right| < \gamma \ (0 < \gamma \le 1;\ z \in \mathbb{U}) \right\};$$

(iii): $\mathcal{S}(f, \frac{z}{1-z}; A, B; \alpha, \beta) = \mathcal{S}S^*(f; A, B; \alpha, \beta)$

$$\left|\frac{\frac{zf'(z)}{f(z)} - \beta\left|\frac{zf'(z)}{f(z)} - 1\right| - 1}{B\left[\frac{zf'(z)}{f(z)} - \beta\left|\frac{zf'(z)}{f(z)} - 1\right|\right] - [B + (A-B)(1-\alpha)]}\right| < 1;$$

(iv): $\mathcal{S}(f, \frac{z}{(1-z)^2}; A, B; \alpha, \beta) = \mathcal{S}K(f; A, B; \alpha, \beta)$

$$\left|\frac{\frac{zf''(z)}{f'(z)} - \beta\left|\frac{zf''(z)}{f'(z)} - 1\right| - 1}{B\left[\frac{zf''(z)}{f'(z)} - \beta\left|\frac{zf''(z)}{f'(z)} - 1\right|\right] - [B + (A-B)(1-\alpha)]}\right| < 1;$$

(v): $\mathcal{S}(f, \frac{z}{1-z}; 1, -1; \alpha, \beta) = \beta - ST(\alpha)$ *and* $\mathcal{S}(f, \frac{z}{(1-z)^2}; 1, -1; \alpha, \beta) = \beta - UCV(\alpha)$ *(see Kanas and Wisniowska [4, 5]);*

(vi): $\mathcal{S}(f, \frac{z}{1-z}; 1, -1; \alpha, 0) = \mathcal{S}^*(\alpha)$ *and* $\mathcal{S}(f, \frac{z}{(1-z)^2}; 1, -1; \alpha, 0) = K(\alpha)$ *(see Silverman [15]).*

Definition 1.3. *For $\delta < 1$ and $|\theta| \le \frac{\pi}{2}$, we define the class $\mathcal{R}(\theta, \delta)$ which consists of functions $g(z)$ of the form (2) and satisfying the condition:*

$$\Re[e^{i\theta}(g'(z) - \delta)] > 0 \ (z \in \mathbb{U}).$$

Clearly, we have $\mathcal{R}(\theta,\delta) \subset \mathcal{S}\,(0 \le \delta < 1)$. *Furthermore, if the function* $g(z)$ *of the form (2) belongs to the class* $\mathcal{R}(\theta,\delta)$, *then*

$$|b_n| \le \frac{2(1-\delta)\cos\theta}{n}\,(n \ge 2).$$

The class $\mathcal{R}(\theta,\delta)$ *studied by Kanas and srivastava* [3].

Very recently, Porwal [9] introduce a power series whose coefficients are probabilities of Poisson distribution:

$$\mathcal{H}(m;z) = z + \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} z^n, \quad (z \in \mathbb{U}). \tag{3}$$

We note that, by ratio test, the radius of convergence of the above series is infinity. Also, we define the function

$$\begin{aligned}\psi(m,\mu;z) &= (1-\mu)\mathcal{H}(m;z) + \mu z(\mathcal{H}(m;z))' \\ &= z + \sum_{n=2}^{\infty}[1+\mu(n-1)]\frac{m^{n-1}}{(n-1)!}e^{-m}z^n\,(\mu \ge 0).\end{aligned}$$

Also, we define the linear operator $\mathcal{T}_m(f*g) : \mathcal{A} \to \mathcal{A}$ by the convolution as

$$[\mathcal{T}_m(f*g)](z) = [\mathcal{H}(m;z)] * [(f*g)(z)] = z + \sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}a_n b_n e^{-m} z^n,$$

and the linear operator $\mathcal{P}_m(f*g) : \mathcal{A} \to \mathcal{A}$ by the integral convolution as

$$[\mathcal{P}_m(f*g)](z) = [\mathcal{H}(m;z)] \circledast [(f*g)(z)] = z + \sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}\frac{a_n b_n}{n} e^{-m} z^n.$$

Also, we define the linear operator $K_\mu(f*g) : \mathcal{A} \to \mathcal{A}$ by the convolution as

$$[K_\mu(f*g)](z) = [\psi(\mu,m;z)] * [(f*g)(z)] = z + \sum_{n=2}^{\infty}[1+\mu(n-1)]\frac{m^{n-1}}{(n-1)!}a_n b_n e^{-m} z^n,$$

and the linear operator $\mathcal{N}_\mu(f*g) : \mathcal{A} \to \mathcal{A}$ by the integral convolution as

$$[\mathcal{N}_\mu(f*g)](z) = [\psi(\mu,m;z)] \circledast [(f*g)(z)] = z + \sum_{n=2}^{\infty}[1+\mu(n-1)]\frac{m^{n-1}}{(n-1)!}\frac{a_n b_n}{n} e^{-m} z^n.$$

The main objective of this paper is to obtain sufficient conditions for the power series with coefficients are the probabilities of Poisson distribution given by (3) and some related series to be in various subclasses of analytic functions.

2. Main Results

Unless otherwise mentioned, we assume throughout this paper that $0 \le \alpha < 1$, $\beta \ge 0$, $\delta < 1$, $-1 \le B < A \le 1$, $|\theta| \le \frac{\pi}{2}$ and $m > 0$. To establish our results, we need the following Lemma.

Lemma 2.1. ([2], Theorem 1) *A sufficient condition for* $f(z)$ *defined by (1) to be in the class* $\mathcal{S}(f,g;A,B;\alpha,\beta)$ *is*

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1) + (A-B)(1-\alpha)]|a_n b_n| \le (A-B)(1-\alpha).$$

Theorem 2.1. *If $g(z) \in \mathcal{R}(\theta, \delta)$ and the inequality*

$$(1-B)(1+\beta)(1-e^{-m})+[(A-B)(1-\alpha)-(1-B)(1+\beta)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq \frac{(A-B)(1-\alpha)}{2(1-\delta)\cos\theta}, \tag{4}$$

satisfied, then $\mathcal{H}(m; z)$ is in the class $\mathcal{S}(\mathcal{H}, g; A, B; \alpha, \beta)$.

Proof. Let $g(z)$ of the form (2) belong to the class $\mathcal{R}(\theta, \delta)$. According to Lemma 2.1, we need only prove that

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)]|b_n|\frac{m^{n-1}}{(n-1)!}e^{-m} \leq (A-B)(1-\alpha).$$

Thus

$$\begin{aligned}
T &= \sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)]\frac{m^{n-1}}{(n-1)!}e^{-m}\frac{2(1-\delta)\cos\theta}{n}, \\
&= 2(1-\delta)\cos\theta e^{-m}\left[(1-B)(1+\beta)\sum_{n=2}^{\infty}(n-1)\frac{m^{n-1}}{n!}+(A-B)(1-\alpha)\sum_{n=2}^{\infty}\frac{m^{n-1}}{n!}\right], \\
&= 2(1-\delta)\cos\theta e^{-m}\left[(1-B)(1+\beta)\left\{\sum_{n=1}^{\infty}\frac{m^n}{n!}-\frac{1}{m}\sum_{n=2}^{\infty}\frac{m^n}{n!}\right\}+(A-B)(1-\alpha)\frac{1}{m}\sum_{n=2}^{\infty}\frac{m^n}{n!}\right], \\
&= 2(1-\delta)\cos\theta e^{-m}\left[(1-B)(1+\beta)(e^m-1)+\{(A-B)(1-\alpha)-(1-B)(1+\beta)\}\frac{1}{m}(e^m-1-m)\right], \\
&= 2(1-\delta)\cos\theta\left[(1-B)(1+\beta)(1-e^{-m})+[(A-B)(1-\alpha)-(1-B)(1+\beta)]\frac{1}{m}(1-e^{-m}-me^{-m})\right],
\end{aligned}$$

and the last expression is bounded above by $(A-B)(1-\alpha)$ if (4) holds. This completes the proof of Theorem 2.1. □

Corollary 2.1. *Let $\beta = 0$, in Theorem 2.1, then $\mathcal{H}(m; z)$ is in the class $\mathcal{S}(\mathcal{H}, g; A, B; \alpha)$, if the inequality*

$$(1-B)(1-e^{-m})+[(A-B)(1-\alpha)-(1-B)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq \frac{(A-B)(1-\alpha)}{2(1-\delta)\cos\theta},$$

is satisfied.

Corollary 2.2. *Let $\beta = 0$, $A = \gamma$ and $B = -\gamma$ in Theorem 2.1, then $\mathcal{H}(m; z)$ is in the class $\mathcal{S}(\mathcal{H}, g; \gamma, \alpha)$, if the inequality*

$$(1+\gamma)(1-e^{-m})+[2\gamma(1-\alpha)-(1+\gamma)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq \frac{\gamma(1-\alpha)}{(1-\delta)\cos\theta},$$

is satisfied.

Remark 2.1.

- **(i):** Let $A = 1$, and $B = -1$ in Theorem 2.1, we give the result obtained by Srivastava and Porwal [16] Theorem 2.2 with $|\tau| = \cos\theta$ and $\gamma = 1$.
- **(ii):** Let $\gamma = 1$ in Corollary 2.2, we give the result obtained by Porwal and Kumar [11] Theorem 3.2 with $|\tau| = \cos\theta$, $A = 1 - 2\delta$, $\lambda = 0$ and $B = -1$.

Theorem 2.2. *If $g(z) \in \mathcal{R}(\theta, \delta)$ and the inequality*

$$[(1-\mu)(1-B)(1+\beta)+\mu(A-B)(1-\alpha)](1-e^{-m})+[(1-\mu)(A-B)(1-\alpha) - (1-\mu)(1-B)(1+\beta)]\frac{1}{m}(1-e^{-m}-me^{-m})+\mu m(1-B)(1+\beta) \leq \frac{(A-B)(1-\alpha)}{2(1-\delta)\cos\theta}, \tag{5}$$

satisfied, then $\psi(\mu, m; z)$ is in the class $\mathcal{S}(\psi, g; A, B; \alpha, \beta)$.

Proof. Let $g(z)$ of the form (2) belong to the class $\mathcal{R}(\theta, \delta)$. According to Lemma 2.1, we need only prove that

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)][1+\mu(n-1)]|b_n|\frac{m^{n-1}}{(n-1)!}e^{-m} \leq (A-B)(1-\alpha).$$

Thus,

$$\begin{aligned}
T1 &= \sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)][1+\mu(n-1)]\frac{m^{n-1}}{(n-1)!}e^{-m}\frac{2(1-\delta)\cos\theta}{n}, \\
&= 2(1-\delta)\cos\theta e^{-m}[\{(1-B)(1+\beta)+\mu(A-B)(1-\alpha)\}\sum_{n=2}^{\infty}(n-1)\frac{m^{n-1}}{n!} \\
&\quad + \mu(1-B)(1+\beta)\sum_{n=2}^{\infty}(n^2-2n+1)\frac{m^{n-1}}{n!}+(A-B)(1-\alpha)\sum_{n=2}^{\infty}\frac{m^{n-1}}{n!}], \\
&= 2(1-\delta)\cos\theta e^{-m}[\{(1-\mu)(1-B)(1+\beta)+\mu(A-B)(1-\alpha)\}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!} \\
&\quad + \{(1-\mu)(A-B)(1-\alpha)-(1-\mu)(1-B)(1+\beta)\}\frac{1}{m}\sum_{n=2}^{\infty}\frac{m^n}{n!}+\mu m(1-B)(1+\beta)\sum_{n=2}^{\infty}\frac{m^{n-2}}{(n-2)!}],
\end{aligned}$$

$$\begin{aligned}
&= 2(1-\delta)\cos\theta e^{-m}[\{(1-\mu)(1-B)(1+\beta)+\mu(A-B)(1-\alpha)\}\sum_{n=1}^{\infty}\frac{m^n}{n!} \\
&+ \{(1-\mu)(A-B)(1-\alpha)-(1-\mu)(1-B)(1+\beta)\}\frac{1}{m}\sum_{n=2}^{\infty}\frac{m^n}{n!}+\mu m(1-B)(1+\beta)\sum_{n=0}^{\infty}\frac{m^n}{n!}], \\
&= 2(1-\delta)\cos\theta[(1-\mu)(1-B)(1+\beta)+\mu(A-B)(1-\alpha)](1-e^{-m})+[(1-\mu)(A-B)(1-\alpha) \\
&- (1-\mu)(1-B)(1+\beta)]\frac{1}{m}(1-e^{-m}-me^{-m})+\mu m(1-B)(1+\beta),
\end{aligned}$$

and the last expression is bounded above by $(A-B)(1-\alpha)$ if (5) holds. This completes the proof of Theorem 2.2. □

Corollary 2.3. *Let $\beta = 0$ in Theorem 2.2, then $\psi(\mu, m; z)$ is in the class $\mathcal{S}(\psi, g; A, B; \alpha)$, if the inequality*

$$[(1-\mu)(1-B)+\mu(A-B)(1-\alpha)](1-e^{-m})+[(1-\mu)(A-B)(1-\alpha) - (1-\mu)(1-B)]\frac{1}{m}(1-e^{-m}-me^{-m})+\mu m(1-B) \leq \frac{(A-B)(1-\alpha)}{2(1-\delta)\cos\theta},$$

is satisfied.

Corollary 2.4. *Let $\beta = 0$, $A = \gamma$ and $B = -\gamma$ in Theorem 2.2, then $\psi(\mu, m; z)$ is in the class $\mathcal{S}(\psi, g; \gamma, \alpha)$, if the inequality*

$$[(1-\mu)(1+\gamma) + 2\gamma\mu(1-\alpha)](1-e^{-m}) + [2\gamma(1-\mu)(1-\alpha) - (1-\mu)(1+\gamma)]\frac{1}{m}(1-e^{-m}-me^{-m}) + \mu m(1+\gamma) \leq \frac{\gamma(1-\alpha)}{(1-\delta)\cos\theta},$$

is satisfied.

Remark 2.2. *Let $\gamma = 1$ in Corollary 2.4, we give the result obtained by Porwal and Kumar* [11] *Theorem 3.2 with $|\tau| = \cos\theta$, $A = 1-2\delta$, $B = -1$ and $\lambda = 0$.*

Theorem 2.3. *If the inequality*

$$e^m\left[(1-B)(1+\beta)m^2 + [(A-B)(1-\alpha) + 2(1-B)(1+\beta)]m\right] \leq (A-B)(1-\alpha), \tag{6}$$

is true, then $[\mathcal{T}_m(f*g)](z)$ maps $(f*g) \in \mathcal{S}$ (or $\mathcal{S}^*$) to $\mathcal{S}(f, g; A, B; \alpha, \beta)$.

Proof. According to Lemma 2.1, we need only prove that

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1) + (A-B)(1-\alpha)]|a_nb_n|\frac{m^{n-1}}{(n-1)!}e^{-m} \leq (A-B)(1-\alpha),$$

and using the fact $|a_nb_n| \leq n$ for $(f*g) \in \mathcal{S}$ (or $\mathcal{S}^*$), then

$$\begin{aligned}
T2 &= \sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1) + (A-B)(1-\alpha)]n\frac{m^{n-1}}{(n-1)!}e^{-m},\\
&= (1-B)(1+\beta)e^{-m}\sum_{n=2}^{\infty}(n-1)(n-2)\frac{m^{n-1}}{(n-1)!}\\
&+ [2(1-B)(1+\beta) + (A-B)(1-\alpha)]e^{-m}\sum_{n=2}^{\infty}(n-1)\frac{m^{n-1}}{(n-1)!} + (A-B)(1-\alpha)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!},\\
&= (1-B)(1+\beta)m^2e^{-m}\sum_{n=0}^{\infty}\frac{m^n}{n!} + [2(1-B)(1+\beta) + (A-B)(1-\alpha)]me^{-m}\sum_{n=0}^{\infty}\frac{m^n}{n!}\\
&+ (A-B)(1-\alpha)e^{-m}\left[\sum_{n=0}^{\infty}\frac{m^n}{(n)!} - 1\right],\\
&= (1-B)(1+\beta)(m^2+2m) + (A-B)(1-\alpha)(m+1-e^{-m}),
\end{aligned}$$

and the last expression is bounded above by $(A-B)(1-\alpha)$ if (6) holds. This completes the proof of Theorem 2.3. □

Corollary 2.5. *Let $\beta = 0$ in Theorem 2.3, then $[\mathcal{T}_m(f*g)](z)$ maps $(f*g) \in \mathcal{S}$ (or $\mathcal{S}^*$) to $\mathcal{S}(f, g; A, B; \alpha)$, if the inequality*

$$e^m\left[(1-B)m^2 + [(A-B)(1-\alpha) + 2(1-B)]m\right] \leq (A-B)(1-\alpha),$$

is true.

Corollary 2.6. *Let $\beta = 0$, $A = \gamma$ and $B = -\gamma$ in Theorem 2.3, then $[\mathcal{T}_m(f*g)](z)$ maps $(f*g) \in \mathcal{S}$ (or $\mathcal{S}^*$) to $\mathcal{S}(f, g; \gamma; \alpha)$, if the inequality*

$$e^m\left[(1+\gamma)m^2 + [2\gamma(1-\alpha) + 2(1+\gamma)]m\right] \leq 2\gamma(1-\alpha),$$

is true.

Theorem 2.4. *If the inequality*

$$(1-B)(1+\beta)e^m m \leq (A-B)(1-\alpha), \tag{7}$$

is true, then

(1) $[\mathcal{T}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;A,B;\alpha,\beta)$,
(2) $[\mathcal{P}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{S}$ *(or* $\mathcal{S}^*$*) to* $\mathcal{S}(f,g;A,B;\alpha,\beta)$.

Proof. 1. According to Lemma 2.1, we need only prove that

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)]|a_n b_n|\frac{m^{n-1}}{(n-1)!}e^{-m} \leq (A-B)(1-\alpha),$$

and using the fact $|a_n b_n| \leq 1$ for $(f*g) \in \mathcal{K}$, then

$$\begin{aligned}
l &= \sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)]\frac{m^{n-1}}{(n-1)!}e^{-m}, \\
&= (1-B)(1+\beta)e^{-m}\sum_{n=2}^{\infty}(n-1)\frac{m^{n-1}}{(n-1)!}+(A-B)(1-\alpha)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}, \\
&= (1-B)(1+\beta)me^{-m}\sum_{n=2}^{\infty}\frac{m^{n-2}}{(n-2)!}+(A-B)(1-\alpha)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}, \\
&= (1-B)(1+\beta)me^{-m}\sum_{n=0}^{\infty}\frac{m^{n}}{n!}+(A-B)(1-\alpha)e^{-m}\left[\sum_{n=0}^{\infty}\frac{m^{n}}{n!}-1\right], \\
&= (1-B)(1+\beta)m+(A-B)(1-\alpha)[1-e^{-m}],
\end{aligned}$$

and the last expression is bounded above by $(A-B)(1-\alpha)$ if (7) holds.
2. The proof is similar to above with using the fact that $|a_n b_n| \leq n$, so we omit it. This completes the proof of Theorem 2.4. □

Corollary 2.7. *Let* $\beta = 0$ *in Theorem 2.4. If the inequality*

$$(1-B)me^m \leq (A-B)(1-\alpha),$$

is true, then

(1) $[\mathcal{T}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;A,B;\alpha)$,
(2) $[\mathcal{P}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{S}$ *(or* $\mathcal{S}^*$*) to* $\mathcal{S}(f,g;A,B;\alpha)$.

Corollary 2.8. *Let* $\beta = 0$, $A = \gamma$, $B = -\gamma$ *in Theorem 2.4. If the inequality*

$$(1+\gamma)me^m \leq 2\gamma(1-\alpha),$$

is true, then

(1) $[\mathcal{T}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;\gamma;\alpha)$,
(2) $[\mathcal{P}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{S}$ *(or* $\mathcal{S}^*$*) to* $\mathcal{S}(f,g;\gamma;\alpha)$.

Remark 2.3.

- **(I):** Let $g(z) = \frac{z}{1-z}$ and $\gamma = 1$ in Corallary 2.8, we give the result obtained by Porwal [9] Theorem 3 with $\lambda = 0$,
- **(II):** Let $g(z) = \frac{z}{1-z}$ and $\gamma = 1$ in Corallary 2.8, we give the result obtained by Porwal and Kumar [11] Theorem 2.1 with $\lambda = 0$.

Theorem 2.5. *If the inequality*

$$(1-B)(1+\beta)(1-e^{-m})+[(A-B)(1-\alpha)-(1-B)(1+\beta)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq (A-B)(1-\alpha), \tag{8}$$

is true, then $[\mathcal{P}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;A,B;\alpha,\beta)$.

Proof. According to Lemma 2.1, we need only prove that

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)]\frac{|a_nb_n|}{n}\frac{m^{n-1}}{(n-1)!}e^{-m} \leq (A-B)(1-\alpha),$$

and using the fact $|a_nb_n| \leq 1$ for $(f*g) \in \mathcal{K}$, then

$$\begin{aligned}
T3 &= \sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)]\frac{m^{n-1}}{n!}e^{-m},\\
&= (1-B)(1+\beta)e^{-m}\sum_{n=2}^{\infty}n\frac{m^{n-1}}{n!} + [(A-B)(1-\alpha)-(1-B)(1+\beta)]e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{n!},\\
&= (1-B)(1+\beta)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!} + [(A-B)(1-\alpha)-(1-B)(1+\beta)]e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{n!},\\
&= (1-B)(1+\beta)e^{-m}\sum_{n=1}^{\infty}\frac{m^{n}}{n!} + [(A-B)(1-\alpha)-(1-B)(1+\beta)]\frac{e^{-m}}{m}\sum_{n=2}^{\infty}\frac{m^{n}}{n!},\\
&= (1-B)(1+\beta)e^{-m}\left[\sum_{n=0}^{\infty}\frac{m^{n}}{n!}-1\right] + [(A-B)(1-\alpha)-(1-B)(1+\beta)]\frac{e^{-m}}{m}\left[\sum_{n=0}^{\infty}\frac{m^{n}}{n!}-1-m\right],\\
&= (1-B)(1+\beta)[1-e^{-m}] + [(A-B)(1-\alpha)-(1-B)(1+\beta)]\frac{1}{m}[1-e^{-m}-me^{-m}],
\end{aligned}$$

and the last expression is bounded above by $(A-B)(1-\alpha)$ if (8) holds. This completes the proof of Theorem 2.5. □

Corollary 2.9. *Let* $\beta = 0$ *in Theorem 2.5. If the inequality*

$$(1-B)(1-e^{-m})+[(A-B)(1-\alpha)-(1-B)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq (A-B)(1-\alpha),$$

is true, then $[\mathcal{P}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;A,B;\alpha)$.

Corollary 2.10. *Let* $\beta = 0$, $A = \gamma$, $B = -\gamma$ *in Theorem 2.5. If the inequality*

$$(1+\gamma)(1-e^{-m})+[2\gamma(1-\alpha)-(1+\gamma)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq 2\gamma(1-\alpha),$$

is true, then $[\mathcal{P}_m(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;\gamma;\alpha)$.

Theorem 2.6. *If the inequality*

$$\begin{aligned}
&e^m[\mu(1-B)(1+\beta)m^3+[\mu(A-B)(1-\alpha)+(4\mu+1)(1-B)(1+\beta)]m^2\\
&\qquad +[2(\mu+1)(1-B)(1+\beta)+(2\mu+1)(A-B)(1-\alpha)]m] \leq (A-B)(1-\alpha),
\end{aligned} \tag{9}$$

is true, then $[K_\mu(f*g)](z)$ *maps* $(f*g) \in \mathcal{S}$ *(or* $\mathcal{S}^*$*) to* $\mathcal{S}(f,g;A,B;\alpha,\beta)$.

Proof. According to Lemma 2.1, we need only prove that

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)][1-\mu(n-1)]|a_nb_n|\frac{m^{n-1}}{(n-1)!}e^{-m} \le (A-B)(1-\alpha),$$

and using the fact $|a_nb_n| \le n$ for $(f*g) \in \mathcal{S}$ (or $\mathcal{S}^*$), then

$$\begin{aligned}
T4 &= \sum_{n=2}^{\infty} n[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)][1-\mu(n-1)]\frac{m^{n-1}}{(n-1)!}e^{-m},\\
&= \mu(1-B)(1+\beta)e^{-m}\sum_{n=2}^{\infty}(n-1)(n-2)(n-3)\frac{m^{n-1}}{(n-1)!} + [\mu(A-B)(1-\alpha)\\
&\quad + (4\mu+1)(1-B)(1+\beta)]e^{-m}\sum_{n=2}^{\infty}(n-1)(n-2)\frac{m^{n-1}}{(n-1)!} + [2(\mu+1)(1-B)(1+\beta)\\
&\quad + (1+2\mu)(A-B)(1-\alpha)]e^{-m}\sum_{n=2}^{\infty}(n-1)\frac{m^{n-1}}{(n-1)!} + (A-B)(1-\alpha)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!},\\
&= \mu(1-B)(1+\beta)m^3e^{-m}\sum_{n=4}^{\infty}\frac{m^{n-4}}{(n-4)!} + [\mu(A-B)(1-\alpha)\\
&\quad + (4\mu+1)(1-B)(1+\beta)]m^2e^{-m}\sum_{n=3}^{\infty}\frac{m^{n-3}}{(n-3)!} + [2(\mu+1)(1-B)(1+\beta)\\
&\quad + (1+2\mu)(A-B)(1-\alpha)]me^{-m}\sum_{n=2}^{\infty}\frac{m^{n-2}}{(n-2)!} + (A-B)(1-\alpha)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!},
\end{aligned}$$

$$= \mu(1-B)(1+\beta)m^3e^{-m}\sum_{n=0}^{\infty}\frac{m^n}{n!} + [\mu(A-B)(1-\alpha)+(4\mu+1)(1-B)(1+\beta)]m^2e^{-m}$$

$$\sum_{n=0}^{\infty}\frac{m^n}{n!} + [2(\mu+1)(1-B)(1+\beta)+(1+2\mu)(A-B)(1-\alpha)]me^{-m}\sum_{n=0}^{\infty}\frac{m^n}{n!}$$

$$+ (A-B)(1-\alpha)e^{-m}\left[\sum_{n=0}^{\infty}\frac{m^n}{n!}-1\right],$$

$$= \mu(1-B)(1+\beta)m^3 + [\mu(A-B)(1-\alpha)+(4\mu+1)(1-B)(1+\beta)]m^2$$

$$+ [2(\mu+1)(1-B)(1+\beta)+(1+2\mu)(A-B)(1-\alpha)]m + (A-B)(1-\alpha)[1-e^{-m}],$$

and the last expression is bounded above by $(A-B)(1-\alpha)$ if (9) holds. This completes the proof of Theorem 2.6. □

Corollary 2.11. *Let $\beta = 0$ in Theorem 2.6. If the inequality*

$$\begin{aligned}
e^m[\mu(1-B)m^3 + [\mu(A-B)(1-\alpha)+(4\mu+1)(1-B)]m^2 + [2(\mu+1)(1-B)\\
+ (2\mu+1)(A-B)(1-\alpha)]m] \le (A-B)(1-\alpha),
\end{aligned}$$

is true, then $[K_\mu(f*g)](z)$ maps $(f*g) \in \mathcal{S}$ (or $\mathcal{S}^*$) to $\mathcal{S}(f,g;A,B;\alpha)$.

Corollary 2.12. *Let $\beta = 0$, $A = \gamma$, $B = -\gamma$ in Theorem 2.6. If the inequality*

$$\begin{aligned}
e^m[\mu(1+\gamma)m^3 + [2\mu\gamma(1-\alpha)+(4\mu+1)(1+\gamma)]m^2 + [2(\mu+1)(1+\gamma)\\
+ 2(2\mu+1)\gamma(1-\alpha)]m] \le 2\gamma(1-\alpha),
\end{aligned}$$

is true, then $[K_\mu(f*g)](z)$ maps $(f*g) \in \mathcal{S}$ (or $\mathcal{S}^*$) to $\mathcal{S}(f,g;\gamma;\alpha)$.

Remark 2.4.

(i): Let $g(z)=\frac{z}{(1-z)^2}$ and $\gamma=1$ in Corollary 2.12, we give the result obtained by Porwal and Kumar [11] Theorem 2.2 with $\lambda=\mu$,

(ii): Let $g(z)=\frac{z}{(1-z)^2}$, $A=1$, $B=-1$, and $\mu=1$ in Theorem 2.6, we give the result obtained by Srivastava and Porwal [16] Theorem 2.5.

Theorem 2.7. *If the inequality*

$$e^m[\mu(1-B)(1+\beta)m^2+[\mu(A-B)(1-\alpha)+(1+\mu)(1-B)(1+\beta)]m]\leq(A-B)(1-\alpha), \tag{10}$$

is true, then

(i): $[K_\mu(f*g)](z)$ *maps* $(f*g)\in\mathcal{K}$ *to* $\mathcal{S}(f,g;A,B;\alpha,\beta)$,

(ii): $[\mathcal{N}_\mu(f*g)](z)$ *maps* $(f*g)\in\mathcal{S}$ *(or* $\mathcal{S}^*$*) to* $\mathcal{S}(f,g;A,B;\alpha,\beta)$.

Proof. (i) According to Lemma 2.1, we need only prove that

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)][1+\mu(n-1)]|a_nb_n|\frac{m^{n-1}}{(n-1)!}e^{-m}\leq(A-B)(1-\alpha),$$

and using the fact $|a_nb_n|\leq1$ for $(f*g)\in\mathcal{K}$, then

$$\begin{aligned}
T5 &= \sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)][1+\mu(n-1)]\frac{m^{n-1}}{(n-1)!}e^{-m},\\
&= \mu(1-B)(1+\beta)e^{-m}\sum_{n=2}^{\infty}(n-1)(n-2)\frac{m^{n-1}}{(n-1)!}+[\mu(A-B)(1-\alpha)+(1+\mu)(1+\beta)(1-B)]\\
&\quad e^{-m}\sum_{n=2}^{\infty}(n-1)\frac{m^{n-1}}{(n-1)!}+(A-B)(1-\alpha)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!},\\
&= \mu(1-B)(1+\beta)e^{-m}\sum_{n=3}^{\infty}\frac{m^{n-1}}{(n-3)!}+[\mu(A-B)(1-\alpha)+(1+\mu)(1+\beta)(1-B)]\\
&\quad e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-2)!}+(A-B)(1-\alpha)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!},\\
&= \mu(1-B)(1+\beta)m^2e^{-m}\sum_{n=3}^{\infty}\frac{m^{n-3}}{(n-3)!}+[\mu(A-B)(1-\alpha)+(1+\mu)(1+\beta)(1-B)]\\
&\quad me^{-m}\sum_{n=2}^{\infty}\frac{m^{n-2}}{(n-2)!}+(A-B)(1-\alpha)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!},\\
&= \mu(1-B)(1+\beta)m^2e^{-m}\sum_{n=0}^{\infty}\frac{m^{n}}{n!}+[\mu(A-B)(1-\alpha)+(1+\mu)(1+\beta)(1-B)]\\
&\quad me^{-m}\sum_{n=0}^{\infty}\frac{m^{n}}{n!}+(A-B)(1-\alpha)e^{-m}\sum_{n=1}^{\infty}\frac{m^{n}}{n!},\\
&= \mu(1-B)(1+\beta)m^2+[\mu(A-B)(1-\alpha)+(1+\mu)(1+\beta)(1-B)]m+(A-B)(1-\alpha)(1-e^{-m}),
\end{aligned}$$

and the last expression is bounded above by $(A-B)(1-\alpha)$ if (10) holds.
(ii) The proof is similar to above with using the fact that $|a_nb_n|\leq n$, so we omit it. This completes the proof of Theorem 2.7. $\square$

Corollary 2.13. *Let $\beta = 0$ in Theorem 2.7. If the inequality*

$$e^m[\mu(1-B)m^2 + [\mu(A-B)(1-\alpha) + (1+\mu)(1-B)]m] \leq (A-B)(1-\alpha),$$

is true, then

(i): $[K_\mu(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;A,B;\alpha)$,
(ii): $[\mathcal{N}_\mu(f*g)](z)$ *maps* $(f*g) \in \mathcal{S}$ *(or* $\mathcal{S}^*$*) to* $\mathcal{S}(f,g;A,B;\alpha)$.

Corollary 2.14. *Let $\beta = 0$, $A = \gamma$, $B = -\gamma$ in Theorem 2.7. If the inequality*

$$e^m[\mu(1+\gamma)m^2 + [2\mu\gamma(1-\alpha) + (1+\mu)(1+\gamma)]m] \leq 2\gamma(1-\alpha),$$

is true, then

(i): $[K_\mu(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;\gamma;\alpha)$,
(ii): $[\mathcal{N}_\mu(f*g)](z)$ *maps* $(f*g) \in \mathcal{S}$ *(or* $\mathcal{S}^*$*) to* $\mathcal{S}(f,g;\gamma;\alpha)$.

Remark 2.5.

(I): Let $g(z) = \frac{z}{1-z}$ and $\gamma = 1$ in Corollary 2.14, we give the result obtained by Porwal and Kumar [11] Theorem 2.1,
(II): Let $g(z) = \frac{z}{1-z}$, $A = 1$, $B = -1$, and $\mu = 1$ in Theorem 2.7, we give us the result obtained by Srivastava and Porwal [16] Theorem 2.6.

Theorem 2.8. *If the inequality*

$$\begin{aligned}\mu(1-B)(1+\beta)m &+ [\mu(A-B)(1-\alpha) + (1-\mu)(1-B)(1+\beta)](1-e^{-m}) \\ &+ [(1-\mu)(A-B)(1-\alpha) - (1-\mu)(1-B)(1+\beta)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq (A-B)(1-\alpha),\end{aligned} \tag{11}$$

is true, then $[\mathcal{N}_\mu(f*g)](z)$ *maps* $(f*g) \in \mathcal{K}$ *to* $\mathcal{S}(f,g;A,B;\alpha,\beta)$.

Proof. According to Lemma 2.1, we need only prove that

$$\sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1)+(A-B)(1-\alpha)][1+\mu(n-1)]\left|\frac{a_nb_n}{n}\right|\frac{m^{n-1}}{(n-1)!}e^{-m} \leq (A-B)(1-\alpha),$$

and using the fact $|a_nb_n| \leq 1$ for $(f*g) \in \mathcal{K}$, then

$$\begin{aligned}T6 &= \sum_{n=2}^{\infty}[(1-B)(1+\beta)(n-1) + (A-B)(1-\alpha)][1+\mu(n-1)]\frac{m^{n-1}}{n!}e^{-m}, \\ &= [(1-\mu)(1-B)(1+\beta) + \mu(A-B)(1-\alpha)]e^{-m}\sum_{n=2}^{\infty}n\frac{m^{n-1}}{n!} \\ &+ [(1-\mu)(A-B)(1-\alpha) + (\mu-1)(1+\beta)(1-B)]e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{n!} \\ &+ \mu(1-B)(1+\beta)e^{-m}\sum_{n=2}^{\infty}n(n-1)\frac{m^{n-1}}{n!},\end{aligned}$$

$$= [(1-\mu)(1-B)(1+\beta)+\mu(A-B)(1-\alpha)]e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}$$

$$+ [(1-\mu)(A-B)(1-\alpha)+(\mu-1)(1+\beta)(1-B)]e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{n!}$$

$$+ \mu(1-B)(1+\beta)e^{-m}\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-2)!},$$

$$= [(1-\mu)(1-B)(1+\beta)+\mu(A-B)(1-\alpha)]e^{-m}\sum_{n=1}^{\infty}\frac{m^{n}}{n!}$$

$$+ [(1-\mu)(A-B)(1-\alpha)+(\mu-1)(1+\beta)(1-B)]\frac{e^{-m}}{m}\sum_{n=2}^{\infty}\frac{m^{n}}{n!}$$

$$+ \mu(1-B)(1+\beta)me^{-m}\sum_{n=0}^{\infty}\frac{m^{n}}{n!},$$

$$= [(1-\mu)(1-B)(1+\beta)+\mu(A-B)(1-\alpha)](1-e^{-m})$$

$$+ [(1-\mu)(A-B)(1-\alpha)+(\mu-1)(1+\beta)(1-B)]\frac{1}{m}(1-e^{-m}-me^{-m})+\mu(1-B)(1+\beta)m,$$

and the last expression is bounded above by $(A-B)(1-\alpha)$ if (11) holds. This completes the proof of Theorem 2.8. □

Corollary 2.15. *Let $\beta = 0$ in Theorem 2.8. If the inequality*

$$\mu(1-B)m + [\mu(A-B)(1-\alpha)+(1-\mu)(1-B)](1-e^{-m})$$
$$+ [(1-\mu)(A-B)(1-\alpha)-(1-\mu)(1-B)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq (A-B)(1-\alpha),$$

is true, then $[\mathcal{N}_\mu(f*g)](z)$ maps $(f*g)\in\mathcal{K}$ to $\mathcal{S}(f,g;A,B;\alpha)$.

Corollary 2.16. *Let $\beta = 0$, $A = \gamma$, $B = -\gamma$ in Theorem 2.8. If the inequality*

$$\mu(1+\gamma)m + [2\mu\gamma(1-\alpha)+(1-\mu)(1+\gamma)](1-e^{-m})$$
$$+ [2(1-\mu)\gamma(1-\alpha)-(1-\mu)(1+\gamma)]\frac{1}{m}(1-e^{-m}-me^{-m}) \leq 2\gamma(1-\alpha),$$

is true, then $[\mathcal{N}_\mu(f*g)](z)$ maps $(f*g)\in\mathcal{K}$ to $\mathcal{S}(f,g;\gamma;\alpha)$.

Acknowledgment

The authors are grateful to the referees for their valuable suggestions.

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