Journal of Fractional Calculus and Applications
Vol. 9(1) Jan. 2018, pp.160-166.
ISSN: 2090-5858.
http://fcag-egypt.com/Journals/JFCA/

APPLICATIONS OF TARIG TRANSFORMATION TO NEW FRACTIONAL DERIVATIVES WITH NON SINGULAR KERNEL

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Abstract. In this paper, we have obtained the relation between the new fractional derivatives by using Tarig Transformation along with this as an application; we have solved fractional order partial differential equation by using this new definition.

1. Introduction

The word Transform tells us about shifting the problem into another domain which is simple to calculate rather than the previous domain and after solving it again coming back to the given situation by Inverse Transform . There are different kinds of Transformations having different kernels like Laplace, Fourier, Mellin, Hartley, Yang Fourier and many more to solve the real life problems([6], [8]) mainly in signal processing, computational fluid dynamics, fractals, Bio mathematics and in fractional calculus [6].The fractional calculus has long back history of over centuries but the tremendous growth has done in last 50 years ([6], [8]). Most of the problems in fractional Calculus are solved by using various methods like adomain decomposition method, Iterative Method, Hes variational iteration method, etc. which gives us the approximate solutions to these fractional order Differential Equations along with this some mathematician used to solve these problems by analytical methods [8] also which gives the solutions to these equations much faster than it is done in numerical methods with more accuracy.There are several definitions of fractional order derivative ([6], [8]) which are used to solve real life problems. Recently Abdon Atangana and Dumitru Baleanu [1] gave new definition of fractional derivative with Non local and Non Singular kernel with exponential function, using this definition the fractional order Differential Equation can be solved ([14], [15]) along with these Transforms which are used to solve Differential equations also, in [19] the new Transformation has been defined which has relation with Laplace Transform [19].

The paper is organized as follows:

2010 *Mathematics Subject Classification.* 34A25, 26A33, 33C45, 33E12.
Key words and phrases. Transformation, fractional Derivatives, Kummers Hyper geometric function, Mittag Leffler Function.
Submitted June 22, 2016. Revised may 9, 2017.

In section 2, we have given some known definition of new fractional order derivative and Tarig Transform and its relation with Laplace Transforms with some of its properties. In section 3, we have found the relation between the ABR and ABC ([1], [3]) fractional derivatives by using Tarig Transformation along with an application of both of it we have solved one fractional order partial differential equation by using Tarig Transformation. The last section consists of conclusion part.

2. Definition

Definition 2.1: (Atangana - Baleanu Riemann fractional derivative)
Consider, $f \in H^1(a,b), b > a, \alpha \in [0,1]$ then the new ABR fractional derivative ([1], [3]) of *f(t)* is defined as,

$$ {}^{ABR}{}_aD_t^{\alpha}(f(t)) \;=\; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}[\int_a^t f(s)E_{\alpha}(-\alpha\frac{(t-s)^{\alpha}}{1-\alpha})]\; ds. \tag{1}$$

Definition 2.2: (Atangana - Baleanu Caputo fractional derivative)
Consider, $f \in H^1(a,b), b > a, \alpha \in [0,1]$ then the new ABR fractional derivative ([1], [3]) of *f(t)* is defined as,

$$ {}^{ABC}{}_aD_t^{\alpha}(f(t)) \;=\; \frac{B(\alpha)}{1-\alpha}\Big[\int_a^t f^{'}(s)E_{\alpha}(-\alpha\frac{(t-s)^{\alpha}}{1-\alpha})\Big]\; ds. \tag{2}$$

Definition 2.3: (Mittag - Leffler function)
The Mittag - Leffeler function [2] is an entire function defined by the series

$$ E_{\alpha}(z) \;=\; \sum\nolimits_{k=0}^{\infty}\frac{z^k}{\Gamma(k\alpha+1)}. \tag{3}$$

Definition 2.4: (Tarig Transformation)
Consider, $f \in S$, where the set S is defined as:
$S \;=\; \{f(t)\exists k_1, k_2, |f(t)| < M \cdot e^{\frac{|t|}{k_j}}, t \in (-1)^j \star [0,\infty)\}$ M > 0 a finite number and k_1, k_2 while may be infinite. Then for given function which satisfies the condition of the set S, the Tarig transformation of *f(t)* is defined as [14]

$$ T[f(t)]\xi \;=\; \int_0^{\infty} f(t\xi)e^{\frac{-t}{\xi}}\; dt, \xi \neq 0 \tag{4}$$

(Properties of Tarig Transformation)

a) Linearity Property :
For the function *f(t)* and *g(t)* satisfying the condition of definition (2.3) then the linearity property ([16],[17],[18],[19]) is given by,

$$ T[af(t) + bg(t)](\xi) \;=\; aT[f(t)] + bT[g(t)] \tag{5}$$

b) Convolution Property:
For the function *f(t)* and *g(t)* satisfying the condition of definition (2.3) then the convolution property ([16],[17],[18],[19]) is given by,

$$ T[f \star g](\xi) \;=\; \xi[T[f(t)](\xi)T[g(t)](\xi)] \tag{6}$$

c) Derivative Property:
For the function *f(t)* and *f'(t)* satisfying the condition of definition (2.3) then the derivative property ([16],[17],[18],[19]) is given by,

$$T[f'(t)](\xi) \;=\; \frac{T[f](\xi)}{(\xi)^2} - \frac{1}{\xi}f(0) \tag{7}$$

d) Relation between Laplace and Tarig Transformation:
If a function *f(t)* satisfying the condition of definition (2.3) which is of exponential order with $T[f(t)](\xi) \;=\; F(\xi)$ and $L[f(t)](\xi) \;=\; G(\xi)$ Then
$F(\xi) \;=\; \frac{1}{\xi}G(\frac{1}{\xi^2})$([16],[17],[18])

3. Tarig Transform of New Fractional Derivative with Non - Singular Kernel

3.1. Tarig Transform of Fractional Derivative of a function in ABR sense.
Consider, $f \in H^1(a,b)$ which is of exponential order, then
${}^{ABC}{}_aD_t^\alpha(f(t)) \;=\; {}^{ABR}{}_aD_t^\alpha(f(t))(\xi) + H(t)$ where
$H(t) = \frac{2\sqrt{\pi}B(\alpha)\xi}{(1-\alpha)}f(0)\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{t^{k\alpha-\frac{1}{2}}}{\Gamma(k\alpha+2)}M(k\alpha+1,k\alpha+2,-t)$

Proof $\rightarrow$

(PART A)
Consider function $f \in H^1(a,b)$ which is of exponential order, then the ABR fractional derivative [1],[3] of *f(t)* can be approximated as follows; ABR fractional derivative of a function is given by[1],[3]

$${}^{ABR}{}_aD_t^\alpha(f(t)) \;=\; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}[\int_a^t f(s)E_\alpha(-\alpha\frac{(t-s)^\alpha}{1-\alpha})]\;ds \tag{8}$$

where $f \in H^1(a,b), b > a, \alpha \in [0,1]$ and $B(\alpha)$ is normalization function obeying $B(0) = B(1) = 1$
Now,
${}^{ABR}{}_aD_t^\alpha(f(t)) \;=\; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}[\int_a^t f(x)E_\alpha(-\alpha\frac{(t-x)^\alpha}{1-\alpha})]\;dx$
$= \; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}[\int_0^t f(x)\sum_{k=0}^{\infty}\frac{-((t-x)(\frac{\alpha}{1-\alpha})^{\frac{1}{\alpha}})^{k\alpha}}{\Gamma(k\alpha+1)}]\;dx$
By neglecting the higher order terms, we get
${}^{ABR}{}_aD_t^\alpha(f(t)) \;\approx\; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}[\int_0^t f(x)\sum_{k=0}^{N}\frac{-((t-x)(\frac{\alpha}{1-\alpha})^{\frac{1}{\alpha}})^{k\alpha}}{\Gamma(k\alpha+1)}]\;dx$

$${}^{ABR}{}_aD_t^\alpha(f(t)) \;\approx\; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k[\int_0^t f(x)\frac{(t-x)^{k\alpha}}{\Gamma(k\alpha+1)}]dx \tag{9}$$

Define

$$h(x\text{-}t) \;=\; \begin{cases} (t-x)^{k\alpha}, & 0 < x < t, \\ 0, & otherwise \end{cases} \tag{10}$$

Then the above definition in terms of convolution can be written as,
${}^{ABR}{}_aD_t^\alpha(f(t)) \;=\; \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{1}{\Gamma(k\alpha+1)}\frac{d}{dt}(f\star h)(x)$

By applying Tarig Transformation on both sides and using the property of convolution and derivative, from equation (9) and (10) we get

$T^{ABR}{}_{a}D_t^{\alpha}(f(t))](\xi) = \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{1}{\Gamma(k\alpha+1)\xi^2}[\xi T(f)T(h)] \qquad \xi \neq 0$

$= \frac{B(\alpha)}{1-\alpha}T(f)\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{1}{\Gamma(k\alpha+1)\xi}\int_0^t t^{k\alpha}e^{\frac{-t}{\xi^2}}\ dt\ \xi \neq 0$

$= \frac{B(\alpha)}{1-\alpha}T(f)\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{\xi^{2k\alpha-1}}{\Gamma(k\alpha+1)}\int_0^t(\frac{t}{\xi^2})^{(k\alpha+1)-1}e^{\frac{-t}{\xi^2}}\ dt\ \xi \neq 0$

$= \frac{B(\alpha)}{1-\alpha}T(f)\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\xi^{2k\alpha-2}\frac{1}{\Gamma(k\alpha+2)}M(k\alpha+1, k\alpha+2, -t)$

By using the definition of Kummers Hyper geometric function

(PART B)

Consider function $f \in H^1(a,b)$ which is of exponential order, then the ABC fractional derivative [1],[3] of *f(t)* can be approximated as follows; ABC fractional derivative of a function is given by[1],[3]

${}^{ABC}{}_{a}D_t^{\alpha}(f(t)) = \frac{B(\alpha)}{1-\alpha}[\int_0^t f'(x)E_\alpha(-\alpha\frac{(t-x)^\alpha}{1-\alpha})]\ dx$ where $f \in H^1(a,b), b > a, \alpha \in [0,1]$ and $B(\alpha)$ is normalization function obeying $B(0) = B(1) = 1$

Now,

${}^{ABC}{}_{a}D_t^{\alpha}(f(t)) = \frac{B(\alpha)}{1-\alpha}[\int_0^t f'(x)E_\alpha(-\alpha\frac{(t-x)^\alpha}{1-\alpha})]\ dx$

${}^{ABC}{}_{a}D_t^{\alpha}(f(t)) = \frac{B(\alpha)}{1-\alpha}[\int_0^t f'(x)\sum_{k=0}^{\infty}\frac{(-(\frac{\alpha}{1-\alpha})^{\frac{1}{\alpha}}(t-x))^{k\alpha}}{\Gamma(k\alpha+1)}]\ dx$

$\approx \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}[\int_0^t f'(x)(\frac{\alpha}{1-\alpha})^k\frac{(t-x)^{k\alpha}}{\Gamma(k\alpha+1)}]\ dx$

By neglecting the higher order terms, we get

${}^{ABC}{}_{a}D_t^{\alpha}(f(t)) \approx \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}\left[\int_0^t f'(x)(\frac{\alpha}{1-\alpha})^k\frac{(t-x)^{k\alpha}}{\Gamma(k\alpha+1)}\right]\ dx$

Define

$$h(x\text{-}t) = \begin{cases} (t-x)^{k\alpha}, & 0 < x < t, \\ 0, & otherwise \end{cases} \tag{11}$$

Then the above definition in terms of convolution can be written as,

$${}^{ABC}{}_{a}D_t^{\alpha}(f(t)) = \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}[(\frac{\alpha}{1-\alpha})^k\frac{1}{\Gamma(k\alpha+1)}(f' \star h)(x)] \tag{12}$$

Applying Tarig Transformation on both sides of the equation (12) and using the property of convolution and derivative we get,

$T[{}^{ABC}{}_{a}D_t^{\alpha}(f(t))](\xi) = \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}[(\frac{\alpha}{1-\alpha})^k\frac{1}{\Gamma(k\alpha+1)}[\xi T(f')T(h)]$

$= \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}[(\frac{\alpha}{1-\alpha})^k\frac{1}{\Gamma(k\alpha+1)}\xi([\frac{T(f)}{\xi^2} - \frac{f(0)}{\xi}]\int_0^t t^{k\alpha}e^{\frac{-t}{\xi^2}}\ dt) \qquad \xi \neq 0$

$= \frac{B(\alpha)}{1-\alpha}T(f)\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{\xi^{2k\alpha-2}}{\Gamma(k\alpha+2)}M(k\alpha+1, k\alpha+2, -t) - \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{f(0)}{\Gamma(k\alpha+2)}M(k\alpha+1, k\alpha+2, -t) \qquad \xi \neq 0$

Applying Inverse Tarig Transformation on both sides of the above equation,

${}^{ABC}{}_{a}D_t^{\alpha}(f(t))(\xi) = {}^{ABR}{}_{a}D_t^{\alpha}(f(t))(\xi) + \frac{2\sqrt{\pi}B(\alpha)\xi}{(1-\alpha)}f(0)\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{t^{k\alpha-\frac{1}{2}}}{\Gamma(k\alpha+2)}M(k\alpha+1, k\alpha+2, -t), t \neq 0$ ${}^{ABC}{}_{a}D_t^{\alpha}(f(t))(\xi) = {}^{ABR}{}_{a}D_t^{\alpha}(f(t))(\xi) + H(t), t \neq 0$ where $H(t) = \frac{2\sqrt{\pi}B(\alpha)\xi}{(1-\alpha)}f(0)\sum_{k=0}^{N}(\frac{\alpha}{1-\alpha})^k\frac{t^{k\alpha-\frac{1}{2}}}{\Gamma(k\alpha+2)}M(k\alpha+1, k\alpha+2, -t)$

The ABR and ABC fractional derivative definition has been recently developed and its relation by using Laplace Transform has been already obtained , but the relation between them using Tarig Transform has not been obtained yet. We have found this relation for general case.

4. Applications for solving fractional order Boundary Value Problems

Consider the ABR time fractional Partial differential equation $u_x(x,t) = {u_x}^{\alpha}(x,t)$ with $|x| < \infty$ and $\alpha \in (0,1]$ subject to
Boundary Condition: $u_x \to 0, u(x,t) \to 0$ as $|x| < \infty$ and Initial Condition: $u(0,t) = 1$
Solution: By definition (2.1) we have
$u_x(x,t) \;=\; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\left[\int_a^t u(x,t)E_\alpha(-\alpha\frac{(t-x)^\alpha}{1-\alpha})\right]\,dx$
which can be written in terms of hyper geometric function as follows
$u_x(x,t) \;=\; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\int_a^t u(x,t)[\sum_{k=0}^{\infty}\frac{(-(\frac{\alpha}{1-\alpha})^{\frac{1}{\alpha}}(t-x))^{k\alpha}}{\Gamma(k\alpha+1)}]dx$
By neglecting the higher order terms
$u_x(x,t) \;=\; \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}[\sum_{k=0}^{N}\int_0^t u(x,t)\frac{(t-x)^{k\alpha}}{\Gamma(k\alpha+1)}(\frac{\alpha}{1-\alpha})^k]\;dx$
$u_x(x,t) \;=\; \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}\frac{1}{\Gamma(k\alpha+1)}(\frac{\alpha}{1-\alpha})^k\frac{d}{dt}(u\star h)(x) \qquad (3.5)$
where
Define

$$h(x\text{-}t) \;=\; \begin{cases} (t-x)^{k\alpha}, & 0<x<t, \\ 0, & otherwise \end{cases} \qquad (13)$$

Apply Tarig Transformation to (3.5) an using the properties of convolution ans derivatives with initial condition, we get
$U_x(x,\xi) \;=\; \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}\frac{1}{\Gamma(k\alpha+1)}(\frac{\alpha}{1-\alpha})^k\frac{1}{\xi^2}[\xi U(x,\xi)T(h)]$
$U_x(x,\xi) \;=\; \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}\frac{1}{\Gamma(k\alpha+1)}(\frac{\alpha}{1-\alpha})^k\frac{U(x,\xi)}{\xi}\int_0^t t^{k\alpha}e^{\frac{-t}{\xi^2}}dt \qquad \xi \neq 0$
$U_x(x,\xi) \;=\; \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}\frac{1}{\Gamma(k\alpha+1)}(\frac{\alpha}{1-\alpha})^k\frac{U(x,\xi)}{\xi}\xi^{2k\alpha}\int_0^t(\frac{t}{\xi^2})^{k\alpha}e^{\frac{-t}{\xi^2}}dt$
$\Rightarrow U_x(x,\xi) \;=\; \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{N}\frac{1}{\Gamma(k\alpha+2)}(\frac{\alpha}{1-\alpha})^k\frac{U(x,\xi)}{\xi^3}t^{k\alpha+1}M(k\alpha+1,k\alpha+2,-t)$
To find the solution approximately we take N = 2, so that the above equation becomes
$\Rightarrow U_x(x,\xi) \;=\; \frac{B(\alpha)}{1-\alpha}\sum_{k=0}^{2}\frac{1}{\Gamma(k\alpha+2)}(\frac{\alpha}{1-\alpha})^k\frac{U(x,\xi)}{\xi^3}t^{k\alpha+1}M(k\alpha+1,k\alpha+2,-t)$
$U_x(x,\xi) \;=\; \frac{B(\alpha)}{1-\alpha}\frac{U(x,\xi)}{\xi^3}(\frac{t^{1+\alpha}M(\alpha+1,\alpha+2,-t)}{\Gamma(2+\alpha)}+\frac{t^{1+2\alpha}M(1+2\alpha,2+2\alpha,-t)}{\Gamma(2+2\alpha)}+\frac{tM(1,2,-t)}{\Gamma(2)})$
Define $H(t,\xi) = \frac{B(\alpha)}{1-\alpha}\frac{1}{\xi^3}(\frac{t^{1+\alpha}M(\alpha+1,\alpha+2,-t)}{\Gamma(2+\alpha)}+\frac{t^{1+2\alpha}M(1+2\alpha,2+2\alpha,-t)}{\Gamma(2+2\alpha)}+\frac{tM(1,2,-t)}{\Gamma(2)})$
$\Rightarrow U_x(x,\xi) \;=\; H(t,\xi)U(x,\xi) \qquad \xi \neq 0$
$U(x,\xi) \;=\; Ae^{H(t,\xi)x}$ with $u(0,t) = 1 \Rightarrow U(0,\xi) = \xi$
which gives the equation $U(x,\xi) \;=\; \xi e^{H(t,\xi)x} \qquad \xi \neq 0$
By using the series expansion of $e^X = 1 + X + X^2 + ...$
$\Rightarrow U_x(x,\xi) \;=\; \xi(1 + H(t,\xi)x)$
Applying Inverse Tarig Transformation on both sides of the above equation , we get
$u(x,t) = 1 - \frac{t^{\frac{-3}{2}}}{2\sqrt{\pi}}\frac{\alpha}{(1-\alpha)^2}(\frac{t^{1+\alpha}M(\alpha+1,\alpha+2,-t)}{\Gamma(2+\alpha)}+\frac{t^{1+2\alpha}M(1+2\alpha,2+2\alpha,-t)}{\Gamma(2+2\alpha)}+\frac{tM(1,2,-t)}{\Gamma(2)})x$
$t \neq 0$
which is required analytic solution

5. Conclusion

The aim of this manuscript was to suggest the new approach to solve the fractional partial differential equation by using new definition. To find the solution we use the relation of Mittag - Leffler function and Kummers Hyper geometric functions. This method is useful to find the solution of fractional order partial differential equation with the new definiton since as the value of N increases the error in the solution decreases.

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