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ON CERTAIN SUBCLASS OF MEROMORPHIC FUNCTIONS DEFINED BY CONVOLUTION

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Abstract. For certain meromorphic p-valent functioun ϕ andψ, we study a class of functioun $f(z) = \frac{1}{z^p} + \sum_{n=p}^{\infty} a_n z^n, (a_n \geq 0)$, defined in the punctured unit disc D, satisfying $\Re\left(\frac{(f*\phi)(z)}{(f*\psi)(z)}\right) > \alpha \ \ (0 \leq \alpha \leq 1, z \in D)$. Coeffiicient etimate disortion theorem and radii of starlikeness and convexity are obtained. Further we consider integral operators associated with functions belonging to the aforementioed class.

1. Introduction

Let $\sum_p$denote the class of functions of the from

$$f(z) = \frac{1}{z^p} + \sum_{n=p}^{\infty} a_n z^n \qquad p \in N, N = \{1, 2, 3, ...\} \tag{1}$$

which analytic and p-valent punctured unit disk D=$\{z : 0 < |z| < 1\}$. A function $f \in \sum_p$ is said to be in the class $\Omega_p(\alpha)$ of meromorphic p-valently starlike functions of order α in D if and only if

$$\Re\left\{-\frac{zf'(z)}{f(z)}\right\} > \alpha \qquad (z \in D; 0 \leq \alpha < p; p \in N)$$

Furthermore, a function $f \in \sum_p$ is said to be in the class $A_p(\alpha)$ of meromorphic p-valently convex function of order α in D if and only if

$$\Re\left\{-\left(1 + \frac{zf''(z)}{f'(z)}\right)\right\} > \alpha \qquad (z \in D; 0 \leq \alpha < p; p \in N)$$

The classes $\Omega_p(\alpha)$, $A_p(\alpha)$ and various other subclasses of $\sum_p$ have been studied rather extensively by Frasin and Murugusundaramoorthy [4], Aouf et.al. [1, 2, 3], Joshi and Srivastava [5], Kulkarni et.al. [6], Mogra [8], Owa et.al.[9], Srivastava and Owa [10], Uralegaddi and Somantha [11], and Yang [12] .

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The Hadamard prouct or convolution of the functions $f(z)$ given by (1) and

$$g(z)=\frac{1}{z^p}+\sum_{n=p}^{\infty}b_nz^n$$

is defined by

$$(f*g)(z)=\frac{1}{z^p}+\sum_{n=p}^{\infty}a_nb_nz^n \quad .$$

For two functions f and g analytic in D, we say that the function f is subordinate to g in (denoted by $f\prec g$) if there exists a Schwarz function $w(z)$, analytic in D with $w(0)=0$, and $|w(z)|<1 (z\in D)$, such that $f(z)=g(w(z))$. We introduce here a class $M(\alpha,\phi,\psi)$ which is defined as follows: suppose the functions $\phi(z)$ and $\psi(z)$ are given by

$$\begin{aligned}\phi(z) &= \frac{1}{z^p}+\sum_{n=p}^{\infty}\lambda_nz^n \qquad (2)\\ \psi(z) &= \frac{1}{z^p}+\sum_{n=p}^{\infty}\mu_nz^n \quad .\end{aligned}$$

Where $\mu_n\geq\lambda_n\geq 0 \ (for\ all\ n\geq p)$.

A function $f\in\sum_p$ is said to be in the class $M(\alpha,\phi,\psi)$ if and only if

$$\Re\left\{\frac{(f*\phi)(z)}{(f*\psi)(z)}\right\}>\alpha \qquad (0\leq\alpha<1, z\in D).$$

In the next section we derive suffcient conditions for $f(z)$ to be in the class $M(\alpha,\phi,\psi)$ and $\Omega(\alpha,\phi,\psi)$, which are obtained by using coefficient inequalities .

2. COEFFICIENT INEQUALITIES

Theorem 1 If $f\in\sum_p$ satisfies

$$\sum_{n=p}^{\infty}\{k\mu_n-\lambda_n+|\lambda_n+\mu_n(k-2\alpha)|\}|a_n|\leq 2(1-\alpha) \qquad (3)$$

for same $(0\leq\alpha<1)$, and $(k\geq 1)$, then $f\in M(\alpha,\phi,\psi)$.

Proof Suppose that (3) holds true for α $(0\leq\alpha<1)$ and $\mu_n\geq\lambda_n$ $(k\geq 1)$, for $f\in\sum_p$ it suffices to show that

$$\left|\frac{\frac{(f*\phi)(z)}{(f*\psi)(z)}-k}{\frac{(f*\phi)(z)}{(f*\psi)(z)}+(k-2\alpha)}\right|<1 \qquad (z\in D)$$

we note that

$$\begin{aligned}\left|\frac{\frac{(f*\phi)(z)}{(f*\psi)(z)}-k}{\frac{(f*\phi)(z)}{(f*\psi)(z)}+(k-2\alpha)}\right| &= \left|\frac{k-1+\sum_{n=p}^{\infty}(k\mu_n-\lambda_n)a_nz^{p+n}}{1-2\alpha+k+\sum_{n=p}^{\infty}(\lambda_n+\mu_n(k-2\alpha))a_nz^{p+n}}\right|\\ &\leq \frac{k-1+\sum_{n=p}^{\infty}(k\mu_n-\lambda_n)|a_n|z^{p+n}}{1-2\alpha+k-\sum_{n=p}^{\infty}|\lambda_n+\mu_n(k-2\alpha)||a_n|z^{p+n}}\\ &< \frac{k-1+\sum_{n=p}^{\infty}(k\mu_n-\lambda_n)|a_n|}{1-2\alpha+k-\sum_{n=p}^{\infty}|\lambda_n+\mu_n(k-2\alpha)||a_n|}.\end{aligned}$$

The last experssion is bounded above by 1 if

$$k-1+\sum_{n=p}^{\infty}(k\mu_n-\lambda_n)|a_n|<1-2\alpha+k-\sum_{n=p}^{\infty}|\lambda_n+\mu_n(k-2\alpha)||a_n|$$

which is equivalent to our condition (3) of Theorem 1.
When $\phi(z)=\frac{1}{z^p}-\frac{z^p[p-(p+1)z]}{p(1-z)^2}$ and $\psi(z)=\frac{1}{z^p}+\frac{z^p}{1-z}$ we have $\lambda_n=\frac{-n}{p}$ and $\mu_n=1$ therefor $M(\alpha,\phi,\psi)$ reduse to the class $\Omega_p(\alpha)$. Putting $\lambda_n=\frac{-n}{p}$ and $\mu_n=1$ in Theorem **??**, we obtain the follwing corollary due to Frasin and Murugusundaramoorthy [4].
Corollary 1 Let $\sigma_n(p,k,\alpha)=(p+n+k-1)+|p+n+2\alpha-k-1|$. If $f(z)=\frac{1}{z^p}+\sum_{n=1}^{\infty}a_{n+p-1}z^{p+n-1}$ $(p\in N)$, satisfies $\sigma_n(p,k,\alpha)|a_{p+n-1}|<2(p-\alpha)$ for some $\alpha\,(0\le\alpha<p)$ and some $k\,(k\ge p)$, then $f(z)\in\Omega_p(\alpha)$.
When $k=1$ and $\lambda_n+\mu_n(1-2\alpha)\le 0\le\mu_n-\lambda_n$ we get the following corollary due to Kumar et al [7].
Corollary 2 Let $f(z)\in\sum_p$ given by (1) . If $\sum_{n=1}^{\infty}(\alpha\mu_n-\lambda_n)|a_n|\le 1-\alpha\ for$ $\alpha\ (0\le\alpha<1)\,,\ then\ f(z)\in M_P(g,h,\alpha)\,.$
In view of Theorem 1, we now define the subclass $\Omega(\alpha,\phi,\psi)$ of $M(\alpha,\phi,\psi)$ which consists of functions $f(z)\in\sum_p$ satisfying condition (3) .

3. DISTORTION THEOREM

Theorem 2 If the function $f(z)$ defind by (1) is in the class $\Omega(\alpha,\phi,\psi)$, then for $0<|z|=r<1$, we hava

$$\frac{1}{r^p}-\frac{2(1-\alpha)}{(k\mu_p-\lambda_p)+|\lambda_p+\mu_p(k-2\alpha)|}r^p\le|f(z)|\le\frac{1}{r^p}+\frac{2(1-\alpha)}{(k\mu_p-\lambda_p)+|\lambda_p+\mu_p(k-2\alpha)|}r^p, \tag{4}$$

where the sequence $<\mu_n>$ and $<\mu_n/\lambda_n>$ are nondecreasing.
The bound in (4) is attained for the functions$f(z)$ given by

$$f(z)=\frac{1}{z^p}+\frac{2(1-\alpha)}{(k\mu_p-\lambda_p)+|\lambda_p+\mu_p(k-2\alpha)|}z^p. \tag{5}$$

Proof We observe that the sequences $\langle k\mu_n-\lambda_n\rangle$ and $\langle\lambda_n+\mu_n(k-2\alpha)\rangle$ are nondecreasing. Since $f(z)\in\Omega(\alpha,\phi,\psi)$ from inequality (3) we have

$$\sum_{n=p}^{\infty}|a_n|\le\frac{2(1-\alpha)}{(k\mu_p-\lambda_p)+|\lambda_p+\mu_p(k-2\alpha)|} \tag{6}$$

Thus for $0<|z|=r<1$, and making use of (6) we have

$$\begin{aligned}|f(z)| &\le \left|\frac{1}{z^p}\right|+\sum_{n=p}^{\infty}|a_n||z|^n \qquad (7)\\ &\le \frac{1}{r^p}+r^p\sum_{n=p}^{\infty}|a_n|\\ &\le \frac{1}{r^p}+\frac{2(1-\alpha)}{(k\mu_p-\lambda_p)+|\lambda_p+\mu_p(k-2\alpha)|}r^p\end{aligned}$$

and

$$\begin{aligned} |f(z)| &\geq \left|\frac{1}{z^p}\right| - \sum_{n=p}^{\infty} |a_n| |z|^n \\ &\geq \frac{1}{r^p} - r^p \sum_{n=p}^{\infty} |a_n| \\ &\geq \frac{1}{r^p} - \frac{2(1-\alpha)}{|\lambda_p - k\mu_p| + |\lambda_p + \mu_p (k-2\alpha)|} r^p \end{aligned} \tag{8}$$

which readily yields the inequalitiy (4) .This completes the proof of Theorem 2.

4. RADII OF STARLIKNESS AND CONVEXITY

The radii of starlikeness and convexity for class $M(\alpha, \phi, \psi)$ is given by.

Theorem 3 If the funtion $f(z)$ defind by (1) is in the class $M(\alpha, \phi, \psi)$, then $f(z)$ is meromorphically p-valetly starlike of order $\delta(0 \leq \delta < 1)$ in $|z| < r_1$ where

$$r_1 = \inf \left\{ \frac{(p-\delta)\left[(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|\right]}{2(n+2p-\delta)(1-\alpha)} \right\}^{\frac{1}{n+p}} \tag{9}$$

furthermore, $f(z)$ is meromorphically p-valetly convex of order $\delta(0 \leq \delta < 1)$ in $|z| < r_2$ where

$$r_2 = \inf \left\{ \frac{(p-\delta-2)\left[(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|\right]}{2n[n+2p-\delta](1-\alpha)} \right\}^{\frac{1}{n+p}} \tag{10}$$

The results (9) , (10) are sharp for the function $f(z)$ given by

$$f(z) = \frac{1}{z^p} + \frac{2(k-\alpha)}{(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|} z^{n+p} \quad . \tag{11}$$

Proof To prove (9) it suffices to show that

$$\left| \frac{zf'(z)}{f(z)} + p \right| \leq p - \delta \tag{12}$$

for $|z| < r_1$ we have

$$\begin{aligned} \left| \frac{zf'(z)}{f(z)} + p \right| &= \left| \frac{\sum_{n=p}^{\infty} (n+p) a_n z^{n+p}}{1 + \sum_{n=1}^{\infty} a_n z^{n+p}} \right| \\ &\leq \frac{\sum_{n=p}^{\infty} (n+p) |a_n| |z|^{n+p}}{1 - \sum_{n=p}^{\infty} |a_n| |z|^{n+p}} \quad . \end{aligned} \tag{13}$$

Hence (13) holds true if

$$\sum_{n=p}^{\infty} (n+p) |a_n| |z|^{n+p} \leq (p-\delta) \left[1 - \sum_{n=p}^{\infty} |a_n| |z|^{n+p} \right] \tag{14}$$

that is

$$\sum_{n=p}^{\infty} \frac{(n+2p-\delta)}{(p-\delta)} |a_n| |z|^{n+p} \leq 1. \tag{15}$$

With the aid of (3), (15) is true if

$$|z|^{n+p}\,\frac{(n+2p-\delta)}{(p-\delta)}\leq\frac{k\mu_n-\lambda_n+|\lambda_n+\mu_n\,(k-2\alpha)|}{2\,(1-\alpha)}\qquad n\geq 1. \tag{16}$$

Solving (16) for $|z|$ we obtain

$$|z|\leq\left\{\frac{(p-\delta)\,\{k\mu_n-\lambda_n+|\lambda_n+\mu_n\,(k-2\alpha)|\}}{2(n+2p-\delta)\,(1-\alpha)}\right\}^{\frac{1}{n+p}}\qquad n\geq 1. \tag{17}$$

In precisely the same manner, we can find the radius of convexity asserted by (10) by requiring that

$$\left|\frac{zf''(z)}{f'(z)}+p+1\right|\leq p-\delta. \tag{18}$$

5. CLOSURE THEOREM

Let the functions $f_j(z)$ be defind, for $j\in\{1,2,3,...,m\}$, by

$$f_j(z)=\frac{1}{z^p}+\sum_{n=p}^{\infty}a_{n,j}z^n,\qquad(z\in D)\,. \tag{19}$$

Now, we shall prove the following result for closure of functions in the class $\Omega\,(\alpha,\phi,\psi)$.

Theorem 4 Let the functions $f_j(z),\ j\in\{1,2,3,....m\}$, defined by (19) be in the class $\Omega\,(\alpha,\phi,\psi)$. Then the function $h(z)\in\Omega\,(\alpha,\phi,\psi)$ where

$$h(z)=\sum_{j=p}^{m}b_jf_j(z),\qquad b_j\geq 0\text{ and }\sum_{j=p}^{m}b_j=1. \tag{20}$$

Proof From (20), we can write

$$h(z)=\frac{1}{z^p}+\sum_{n=p}^{\infty}c_nz^n, \tag{21}$$

where

$$c_n=\sum_{j=p}^{m}b_ja_{n,j},\qquad j\in\{1,2,3,....m\}\,. \tag{22}$$

Since $f_j(z)\in\Omega\,(\alpha,\phi,\psi)\,,(\ j\in\{1,2,3,....m\})$, from (3), we have

$$\begin{aligned}&\sum_{n=p}^{\infty}\frac{(k\mu_n-\lambda_n)+|\lambda_n+\mu_n\,(k-2\alpha)|}{2\,(1-\alpha)}\left(\sum_{j=p}^{m}b_ja_{n,j}\right)\\ =&\ \sum_{j=p}^{m}b_j\left(\sum_{n=p}^{\infty}\frac{(k\mu_n-\lambda_n)+|\lambda_n+\mu_n\,(k-2\alpha)|}{2\,(1-\alpha)}a_{n,j}\right)\\ \leq&\ \sum_{j=p}^{m}b_j=1.\end{aligned}$$

Which shows that $h(z)\in\Omega\,(\alpha,\phi,\psi)$ .This completes the proof of Theorem 4.

Theorem 5 Let

$$f_{p-1}=\frac{1}{z^p}\qquad(z\in D) \tag{23}$$

and

$$f_n(z) = \frac{1}{z^p} + \frac{(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|}{2(1-\alpha)} z^n, \tag{24}$$

where $n \geq p$; $z \in D$.Then $f(z) \in \Omega(\alpha, \phi, \psi)$ if and only if it can be expressed in the from

$$f(z) = \sum_{n=p-1}^{\infty} \lambda_n f_n(z), \tag{25}$$

where $\lambda_n \geq 0, (n \in \mathbb{N}_0)$ and $\sum_{n=p-1}^{\infty} \lambda_n = 1$.
Proof From (23), (24), and (25) it is easily see that

$$\begin{aligned} f(z) &= \sum_{n=p-1}^{\infty} \lambda_n f_n(z) \qquad (26) \\ &= \frac{1}{z^p} + \sum_{n=p}^{\infty} \frac{2(1-\alpha)}{(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|} \lambda_n z^n. \end{aligned}$$

Since

$$\sum_{n=p}^{\infty} \frac{(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|}{2(1-\alpha)} . \frac{2(1-\alpha)}{(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|} \lambda_n = \sum_{n=p}^{\infty} \lambda_n = 1 - \lambda_{p-1} \leq 1,$$

it follows from Theorem 1 that the function $f(z)$ given by (25) is in the class $\Omega(\alpha, \phi, \psi)$ .Conversely, let us suppose that $f(z) \in \Omega(\alpha, \phi, \psi)$. Since

$$|a_n| \leq \frac{2(1-\alpha)}{(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|} \qquad (n \geq p).$$

Setting

$$\lambda_n = \frac{(k\mu_n - \lambda_n) + |\lambda_n + \mu_n (k-2\alpha)|}{2(1-\alpha)} |a_n| \qquad (n \geq p)$$

and

$$\lambda_{p-1} = 1 - \sum_{n=p}^{\infty} \lambda_n.$$

It followes that

$$f(z) = \sum_{n=p-1}^{\infty} \lambda_n f_n(z).$$

This completes the proof of the Theorem 5.

6. INTEGRAL OPERATOR

Theorem 6 Let the function $f(z)$defind by (1) be in the class $\Omega(\alpha, \phi, \psi)$and c be a real number such that $c > p$. then the function $F(z)$ defined by

$$F(z) = \frac{c-p}{z^c} \int_0^z t^{c-1} f(t) dt \tag{27}$$

also belongs to the class $\Omega(\alpha, \lambda, g)$.
Proof From the representation of $F(z)$, it followes that

$$F(z) = \frac{1}{z^p} + \sum_{n=p}^{\infty} c_n z^n, \tag{28}$$

where

$$c_n = \left(\frac{c-p}{c+n}\right) a_n. \tag{29}$$

Therefore

$$\begin{aligned} &\sum_{n=p}^{\infty} \left[k\mu_n - \lambda_n + \left|\lambda_n + \mu_n (k-2\alpha)\right|\right] c_n \\ = &\sum_{n=p}^{\infty} \left[k\mu_n - \lambda_n + \left|\lambda_n + \mu_n (k-2\alpha)\right|\right] \left(\frac{c-1}{c+n}\right) a_n \qquad (30) \\ \leq &\sum_{n=2}^{\infty} \left[k\mu_n - \lambda_n + \left|\lambda_n + \mu_n (k-2\alpha)\right|\right] a_n \leq 2(1-\alpha) \end{aligned}$$

sinc $f(z) \in \Omega(\alpha, \phi, \psi)$. Hence, by Theorem 1, $F(z) \in \Omega(\alpha, \phi, \psi)$

Theorem 7 Let c be a real number such that $c > p$. If $F(z) \in \Omega(\alpha, \phi, \psi)$ .Then the function $f(z)$defind by (27) is meromorphically p-valent close-to-convex in $|z| < r^*$, where

$$r^* = \inf_n \left\{ \frac{(p-\alpha)(c-p)\left[(k\mu_n - \lambda_n) + \left|\lambda_n + \mu_n (k-2\alpha)\right|\right]}{2n(c+n)(1-\alpha)} \right\}^{\frac{1}{n+p}} \qquad (n \geq p) \tag{31}$$

The result is sharp .

Proof Let $F(z) = \frac{1}{z^p} + \sum_{n=p}^{\infty} a_n z^n (a_n \geq 0)$. it follows from (27) that

$$f(z) = \frac{z^{1-c}\left[z^c F(z)\right]'}{(c-p)} = \frac{1}{z^p} + \sum_{n=p}^{\infty} \left(\frac{c+n}{c-p}\right) a_n z^n \qquad (c > p). \tag{32}$$

In order to obtain the required result itsuffices to show that $\left|z^{p+1}\ f'(z) + p\right| \leq p - \alpha$ in $|z| < r^*$. Now

$$\left|z^{p+1} f'(z) + p\right| \leq \sum_{n=p}^{\infty} \frac{n(c+n)}{(c-p)} a_n |z|^{n-1}.$$

Thus $\left|z^{p+1} f'(z) + p\right| < p - \alpha$ if

$$\sum_{n=p}^{\infty} \frac{n(c+n)}{(p-\alpha)(c-p)} a_n |z|^{n-1} < 1. \tag{33}$$

Hence by using (3) , (33) will be satisfied if

$$\frac{n(c+n)}{(p-\alpha)(c-p)} |z|^{n+p} < \frac{(k\mu_n - \lambda_n) + \left|\lambda_n + \mu_n (k-2\alpha)\right|}{2(1-\alpha)},$$

i.e, if

$$|z| < \left[\frac{(p-\alpha)(c-p)\left[(k\mu_n - \lambda_n) + \left|\lambda_n + \mu_n (k-2\alpha)\right|\right]}{2n(c+n)(1-\alpha)}\right]^{\frac{1}{n+p}} \qquad (n \geq p). \tag{34}$$

Therefore $f(z)$ is meromorphic close-to-convex in $|z| < r^*$. Sharpness follows if we take

$$f(z) = z + \frac{2n(c+n)(1-\alpha)}{(p-\alpha)(c-p)\left[(k\mu_n - \lambda_n) + \left|\lambda_n + \mu_n (k-2\alpha)\right|\right]} z^n \tag{35}$$

$(n \geq p, c > p)$.

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