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# FRACTIONAL ORDER E-EPIDEMIC MODEL WITH HIGHLY INFECTIOUS NODES

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Abstract. In this paper, SIJR e-epidemic model of fractional order for the transmission of viruses in computer network with natural death has been presented. The fractional derivatives are described in the Caputo sense. In this model the nodes have two levels of infection. Predictor-Corrector method is employed to obtain numerical solution of presented model.

## 1. Introduction

Viruses, worms, Trojans, and bots are all part of a software called malicious code [22]. This code is designed to damage or disrupt networks. In other words, this kind of software causes billions of dollars' worth of economic damage each year. Today in the computer era, e-mails and use of secondary devices are the major responsible sources for the transmission of malicious objects in computer network [20]. More attention has been paid to the e-epidemic models and antivirus countermeasures to study the prevalence of virus [19]. Mathematical modeling of the transmission of infectious diseases have been developed to be adapted to the study of the spread of computer viruses to analyze quantitatively the propagation of computer viruses [11]. In other words, networks and the epidemiology of directly transmitted infectious diseases are fundamentally linked [12]. Based on SIR epidemic model [13-15], dynamical models for malicious objects propagation were proposed [12]. The SEIRS epidemic model was proposed to model the transmission of malicious objects in computer network in [16]. The authors in [21] propose an improved SEI model to simulate virus propagation. SEIQRS model have been developed for the transmission of malicious objects in computer network by Kumar et al in [17]. However, the large amount of such models has been restricted to integer-order ordinary or delay differential equations [16]. Recently, fractional calculus has been extensively applied in many fields [6]. Most of the mathematical theory applicable to the study of non-integer order calculus was developed through the end of $19^{th}$ century. The concept of fractional calculus has tremendous potential to change the way we see the model, and control the nature around us. The major reason of using fractional

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calculus is that, it is naturally related to systems with memory which exists in most biological systems [3-5]. Also, they are closely related to fractals, which are abundant in biological systems [4]. Also factional order differential equations are, at least, as stable as their integer order counterpart [1, 2, 7-10]. Hence, we propose a system of fractional order model which based on the integer model presented in [18] for modeling transmission of viruses in computer network with natural death as follows

$$\begin{aligned} D^{\alpha}(S) &= b - \mu S - \beta SI, \\ D^{\alpha}(I) &= \beta SI + \sigma J - \beta IJ - \gamma I(\mu + \delta)I, \\ D^{\alpha}(J) &= \beta IJ - (\mu + \delta)J - \sigma J + b, \\ D^{\alpha}(R) &= \gamma I - \mu R \end{aligned} \tag{1}$$

Where $0 < \alpha \leq 1$,
$b$ is the birth rate (new nodes attached to the network),
$\mu$ is the natural death rate (that is, crashing of the nodes due to the reason other than the attack of viruses)
$\delta$ is the rate of crashing of the nodes due to the attack of viruses, $\beta$ is the rate coefficient of Susceptible class (for $S$ to $I$) and the rate coefficient of Infectious class (for $I$ to $J$ ),
$\sigma$ is the rate coefficient of highly infectious class (for $J$ to $I$),
$\gamma$ is the rate coefficient of infectious class (for $I$ to $R$).

The authors in [18] evaluate the basic reproduction $R_0$ by using the next generation matrix approach and deduced that $R_0 = \frac{\beta}{\mu+\sigma+\gamma}$. The basic reproduction number $R_0$ is defined as the average number of secondary infections produced by a single infectious host introduced into a totally susceptible population [6]. In most cases, if $R_0 > 1$ then the infection will be able to spread in a population; whereas, if $R_0 < 1$, then the infection will disappear from the population.The rest of the paper is organized as follows. A brief review of the fractional calculus theory is given in Section 2. In section 3, fractional order Equilibrium points and stability are discussed while section 4 is devoted for the numerical results.

## 2. Fractional calculus

Fractional order models have been the focus of many studies due to their frequent appearance in various applications in several scientific fields. We first give the definition of fractional-order integration and fractional-order differentiation [1, 2, 7-10]. For the concept of fractional derivative, we will consider Caputo's definition. It has the advantage of dealing properly with initial value problems.

**Definition 1.** The fractional integral of order $\alpha$ of a function $f : R^{+} \to R$ is given by

$$J^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \quad \alpha > 0, x > 0, \tag{2}$$

$$J^0 f(x) = f(x).$$

Hence we have

$$J^{\alpha}t^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}t^{\alpha+\gamma}, \quad \alpha > 0, \gamma > -1, t > 0 \tag{3}$$

**Definition 2.** Riemann-Liouville and Caputo fractional derivatives of order $\alpha$ where $\alpha \in (n-1, n)$ of a continuous function $f : R^{+} \to R$ is given respectively by

$$D^{\alpha}f(x) = D^{m}(J^{m-\alpha}f(x)), \tag{4}$$

$$D_{*}^{\alpha}f(x) = J^{m-\alpha}(D^{m}f(x)), \tag{5}$$

Where

$$m-1 < \alpha \leq m, m \in N.$$

So it is clear that, calculating time-fractional derivative of a function $f(t)$ at some time $t = t_1$ requires all the previous history, i.e. all $f(t)$ from $t = 0$ to $t = t_1$.

## 3. Equilibrium points and stability

The authors in [18] deduced the equilibrium points of the integer order system of the given model (1), i.e. when $\alpha = 1$ in (1). So we can deduce that, the system (1) has two possible equilibriums, first, the malicious objects free equilibrium, $E_0 = (N, 0, 0, 0)$ and second, the endemic equilibrium $E^{*} = (S^{*}, I^{*}, J^{*}, R^{*})$ where
$S^{*} = \frac{b}{\mu+\beta I^{*}}$
$J^{*} = \frac{b}{\mu+\delta+\sigma-\beta I^{*}}$
$R^{*} = \frac{\gamma I^{*}}{\mu}$
And $I^{*}$ can be obtained from (6)

$$\begin{aligned} I^{*3}\beta^{2}(\mu+\delta+\gamma) - I^{*2}\beta[2b\beta + (\delta+\sigma)(\mu+\delta+\gamma)] \\ +I^{*}[b\beta(\delta+2\sigma) - \mu(\mu+\delta+\sigma)(\mu+\delta+\gamma)] + b\mu\sigma = 0 \end{aligned} \tag{6}$$

A sufficient condition for the local asymptotic stability of the equilibrium points is that the eigenvalues $\lambda_i$ of the Jacobian matrix of $E$ satisfy the condition $|\arg\lambda_i| > \alpha\frac{\pi}{2}$ [1]. This confirms that fractional-order differential equations are, at least, as stable as their integer order counterpart.

## 4. Numerical results

Predictor corrector method is applied to get numerical solutions of (1)

## 5. Conclusion

In this paper, we have introduced fractional order into a compartmental epidemic model with natural death for malicious objects in computer network.The results show that the solution continuously depends on the time-fractional derivative. When $\alpha \to 1$ the solution of the fractional model (1) reduces to the standard solution of the integer order model(fig. (1-10)). It is clear that when the value of natural death rate in creases, the attack of malicious codes increases(fig. (2-5)). Due to run of anti-virus software, infection vanishes (fig.3).

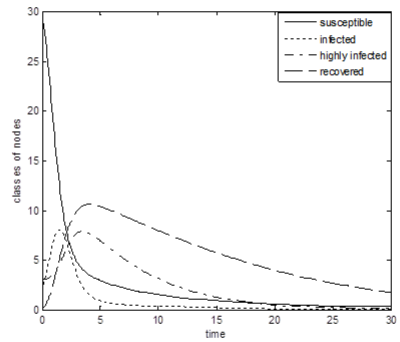


FIGURE 1. Dynamical behavior of the system (1) with the real parameters b = 0.01,  = 0.09, = 0.09, s = 0.09,? = 0.65,d = 0.

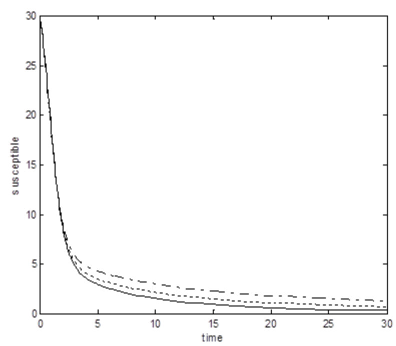


FIGURE 2. The density of fully susceptible nodes when =0.09for a = 1 (solid line), a =0.98 (dashed line), and a = 0.95 (Dashed-dotted).

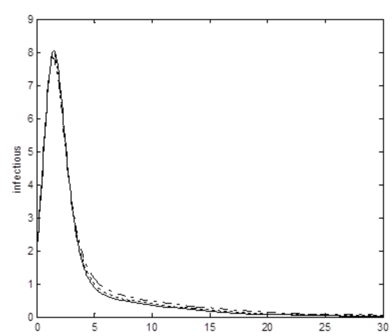


FIGURE 3. The density of infectious nodes with anti-virus software when =0.09 for a = 1 (solid line), a =0.98 (dashed line), and a = 0.95 (Dashed-dotted).

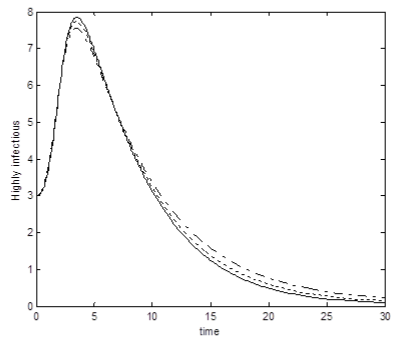


FIGURE 4. The density of highly infectious nodes when =0.09for a = 1 (solid line), a =0.98 (dashed line), and a = 0.95 (Dashed-dotted).

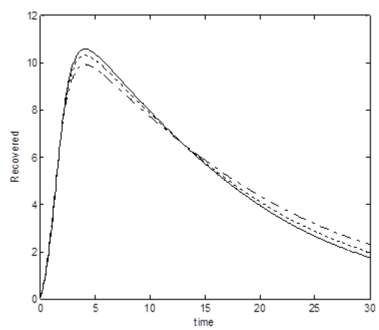


FIGURE 5. The density of recovered nodes when =0.09for a = 1 (solid line), a =0.98 (dashed line), and a = 0.95 (Dashed-dotted).

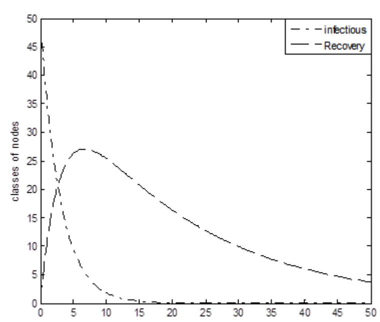


FIGURE 6. Dynamical behavior of the classes I and R when =0.05 ,d=0.03 ,?=0.25

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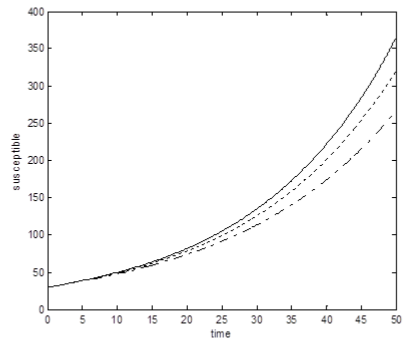


FIGURE 7. The density of fully susceptible nodes when =0.05 for a = 1 (solid line), a =0.98 (dashed line), and a = 0.95 (Dashed-dotted).

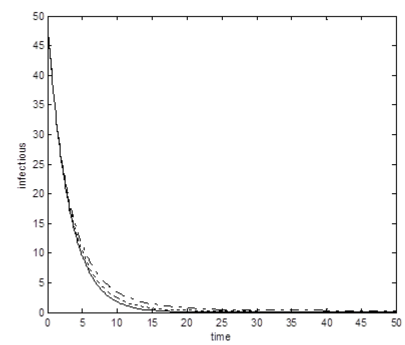


FIGURE 8. The density of infectious when =0.05 for a = 1 (solid line), a =0.98 (dashed line), and a = 0.95 (Dashed-dotted).

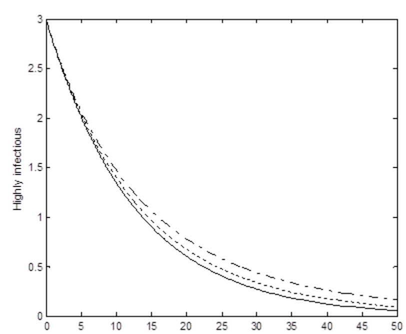


FIGURE 9. The density of highly infectious when =0.05 for a = 1 (solid line), a =0.98 (dashed line), and a = 0.95 (Dashed-dotted).

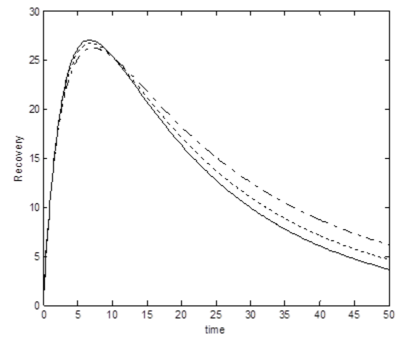


FIGURE 10. The density of recovery when =0.05 for a = 1 (solid line), a =0.98 (dashed line), and a = 0.95 (Dashed-dotted).

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