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LARGE DEFLECTION OF A CIRCULAR PLATE UNDER NON-UNIFORM LOAD PERTAINING TO A PRODUCT OF SPECIAL FUNCTIONS

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ABSTRACT. The main object of this paper is to obtain the large deflection and bending stresses for a clamped circular plate under non-uniform load by using Berger's approximate method. The load shape considered here is an arbitrary function p(x) involving Jacobi polynomial, Fox-Wright function and $\bar{\mathrm{H}}$-functions. The small deflection case is also considered as a particular case of large deflection. The nature of the load shape considered here yields many useful and interesting results while solving the problem. Some known and new results have been evaluated by taking suitable values of parameters.

1. Introduction

The $\bar{\mathrm{H}}$-function given by Inayat-Hussain [6, 7] which is a generalization of the familiar Fox H-function, is as follows

$$\bar{\mathrm{H}}^{\mathrm{M,N}}_{\mathrm{P,Q}}[\mathrm{z}] = \bar{\mathrm{H}}^{\mathrm{M,N}}_{\mathrm{P,Q}}\left[\mathrm{z}\ \middle|\begin{matrix}(\mathrm{a_j},\alpha_\mathrm{j};\mathrm{A_j})_{1,\mathrm{N}},(\mathrm{a_j},\alpha_\mathrm{j})_{\mathrm{N}+1,\mathrm{P}}\\(\mathrm{b_j},\beta_\mathrm{j})_{1,\mathrm{M}},(\mathrm{b_j},\beta_\mathrm{j};\mathrm{B_j})_{\mathrm{M}+1,\mathrm{Q}}\end{matrix}\right]$$
$$= \frac{1}{2\pi \mathrm{i}}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\bar{\phi}(\xi)\,\mathrm{z}^{\xi}\,\mathrm{d}\xi \qquad (1)$$

where

$$\bar{\phi}(\xi) = \frac{\prod_{\mathrm{j}=1}^{\mathrm{M}}\Gamma(\mathrm{b_j}-\beta_\mathrm{j}\xi)\prod_{\mathrm{j}=1}^{\mathrm{N}}\{\Gamma(1-\mathrm{a_j}+\alpha_\mathrm{j}\xi)\}^{\mathrm{A_j}}}{\prod_{\mathrm{j}=\mathrm{M}+1}^{\mathrm{Q}}\{\Gamma(1-\mathrm{b_j}+\beta_\mathrm{j}\xi)\}^{\mathrm{B_j}}\prod_{\mathrm{j}=\mathrm{N}+1}^{\mathrm{P}}\Gamma(\mathrm{a_j}-\alpha_\mathrm{j}\xi)}, \qquad (2)$$
$$\mathrm{i} = \sqrt{(-1)}.$$

This contains the fractional powers of some of the Gamma functions. Here and throughout the paper a_j (j = 1,...,P) and b_j (j = 1,...,Q) are complex parameters $\alpha_\mathrm{j} \geq 0$ (j = 1,...,P), $\beta_\mathrm{j} \geq 0$ (j = 1,...,Q)(not all zero simultaneously) and the experiment A_j (j = 1,...,N) and B_j (j = M+1,...,Q) can take non-integer values.

The contour in (2) is imaginary axis Re(ξ) = 0. It is suitably indented in order to avoid the singularities of the Gamma functions and to keep those singularities on appropriate sides. Again, for A_j (j = 1,...,N) not an integer, the poles of

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the Gamma functions of the numerator in (2) are converted to the branch points. However, as long as there is no coincidence of poles from any $\Gamma(b_j - \beta_j \xi)$ (j = 1,. . . ,.M) and Γ (1− $a_j + \alpha_j$ ξ) (j = 1,. . . ,N) pair, the branch cuts can be chosen so that the path of integration can be distorted in the usual manner. The condition for the absolute convergence of the defining integral for $\bar{H}$-function have been given by Buschman and Srivastava as

$$\Omega = \sum_{j=1}^{M} |B_j| + \sum_{j=1}^{N} A_j \alpha_j - \sum_{j=M+1}^{Q} |B_j \beta_j| - \sum_{j=N+1}^{P} \alpha_j > 0$$

and $|\arg(z)| < \frac{1}{2}\pi\Omega$.

We assume that the convergence and sufficient condition of above function, given by equation (1) is satisfied by each of the various $\bar{H}$-function involved throughout the present work.

The behavior of the $\bar{H}$-function for small values of |z |follows easily from a result recently given by Rathie [[10], p.306], we have

$$\bar{H}^{M,N}_{P,Q}[z] = 0(|z|^{\alpha}), \bar{H}^{M,N}_{P,Q}[z] = 0(|z|^{\alpha}),$$

$$\alpha = \min_{1 \le j \le M} [\mathrm{Re}(b_j/B_j)], \quad |z| \to 0.$$

The series representation of $\bar{H}$-function [[2], p.271] is given by

$$\bar{H}^{S,T}_{U,V}\left[z \left| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,T}, (a'_j,\alpha'_j)_{T+1,U} \\ (b'_j,\beta'_j)_{1,S}, (b'_j,\beta'_j;B'_j)_{S+1,V} \end{matrix} \right.\right] = \sum_{h=1}^{S}\sum_{r=0}^{\infty} \frac{\sigma(s)\,(-1)^r z^s}{r!\ \beta_h} \tag{3}$$

where

$$\sigma(s) = \frac{\prod_{\substack{j=1 \\ j \ne h}}^{S} \Gamma(b'_j - \beta'_j s) \prod_{j=1}^{T} \{\Gamma(1 - a'_j + \alpha'_j s)\}^{A'_j}}{\prod_{j=S+1}^{V} \{\Gamma(1 - b'_j + \beta'_j s)\}^{B_j} \prod_{j=T+1}^{U} \Gamma(a'_j - \alpha'_j s)},$$

$$s = \xi_{h,r} = \frac{b_h + r}{\beta_h}. \tag{4}$$

Also, the Fox-Wright's function [11] is defined as

$${}_{p'}\psi_{q'}(z) = {}_{p'}\psi_{q'}\left[\begin{matrix}(e_j,E_j)_{1,p'}; \\ (f_j,F_j)_{1,q'}\end{matrix}; z\right] = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p'} \Gamma(e_j + E_j n)}{\prod_{j=1}^{q'} \Gamma(f_j + F_j n)} \frac{z^n}{n!}, \tag{5}$$

where $E_j(j = 1, ..., p')$ and $F_j(j = 1, .., q')$are real and positive and

$$1 + \sum_{j=1}^{q'} F_j - \sum_{j=1}^{p'} E_j > 0.$$

Plates are the flat structures whose thickness t is small compared to the other in-plane dimensions. For a circular plate , the only in-plane dimension is the radius ρ.

Plate theories are classified in many ways. One of them is based on the thickness, that is, thin and thick-plate theories. Geometrically, a plate is said to be thin if its thickness ratio t/ρ is less than 1/20, otherwise the plate is known to be thick. The bending properties of a plate depend mainly on its thickness as compared with its other dimensions. There are several theories for plates under large deflection; the most commonly used of them is the Von-Karman plate theory which is sometimes referred to as the Kirchoff-Foppel plate theory.
In the classical theory of plates, small deflection and elastic behavior of the material are assumed. When the lateral deflection exceeds one half the plate thickness [13], the classical theory generally is not adequate and the second order effects of the vertical displacements on the membrane stresses need to be considered. Two-coupled non-linear partial differential equations considering these effects were given by [6]. Solutions based on these differential equations have been known as large deflection solutions. Berger [1] in 1955 proposed an approximate method for investigating the large deflection of initially flat isotropic plates.

Here the large deflection of a clamped circular plate under non-uniform load has been calculated by using Berger's approximate method. We consider the applied external pressure p(x) in the following form:

$$p(x) = K_0\left(1-\frac{x^2}{\rho^2}\right)^{\alpha} P_{\beta}^{a,b}\left(1-\frac{2x^2}{\rho^2}\right) {}_{p'}\psi_{q'}\left\{K_1\left(1-\frac{x^2}{\rho^2}\right)\right\}$$
$$\bar{H}_{P,Q}^{M,N}\left[K_2\left(1-\frac{x^2}{\rho^2}\right)\right] \bar{H}_{U,V}^{S,T}\left[K_3\left(1-\frac{x^2}{\rho^2}\right)\right] \tag{6}$$

where $P_{\beta}^{a,b}(x)$is the Jacobi polynomial [12] and K_0, K_1 and K_2 are constants.

2. Statement of the Problem

Let us assume a clamped circular plate of thickness t, radius ρ and flexural rigidity R. Then by using Berger's method, the approximate equations for a circular plate undergoing large deflections due to an externally applied load p(x) may be given as

$$\left(\frac{d^2}{dx^2}+\frac{1}{x}\frac{d}{dx}\right)\left(\frac{d^2w}{dx^2}+\frac{1}{x}\frac{dw}{dx}-k^2w\right) = \frac{p}{R} = \phi(x) \tag{7}$$

where k is a normalized constant of integration given by the equation

$$\frac{dy}{dx}+\frac{y}{x}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2 = \frac{k^2t^2}{12} \tag{8}$$

where w is the plate deflection, normal to the middle plane of the plate and y is the radial displacement.
The boundary condition of the problem are:
(i) $w = 0 = \frac{dw}{dx}$, at $x = \rho$
(ii) $y = 0$, at $x = \rho$

Solution of the Problem

Let us consider

$$w = \sum_i G_i[J_0(xt_i) - J_0(\rho t_i)] \tag{9}$$

where t_i is the i-th root of $J_1(\rho t_i) = 0$.

It is clear that the boundary conditions are satisfied by the above equation.
Now using (9) in the equation (7), we find

$$\sum_{i} G_i\, t_i^2 (k^2 + t_i^2)\, J_0(xt_i) \;=\; \phi(x) \tag{10}$$

Now expanding $\phi(x)$ in a series of Bessel's function, we obtain on integration

$$\int_0^{\rho} G_i\, t_i^2 (k^2 + t_i^2)\, J_0^2(xt_i)\, x\, dx \;=\; \int_0^{\rho} \phi(x)\; J_0(xt_i)\, \rho\, dx \tag{11}$$

Now by left hand side of (11)

$$\int_0^{\rho} x\, J_0^2(xt_i) dx \;=\; \frac{\rho^2}{2} J_0^2(\rho t_i) \tag{12}$$

(11) becomes

$$G_i\, t_i^2 (k^2 + t_i^2)\, \frac{\rho^2}{2} J_0^2(\rho t_i) = \int_0^{\rho} \phi(x)\; J_0(xt_i)\; x\; dx$$

or

$$G_i = \frac{2 \int_0^{\rho} x\, \phi(x)\; J_0(x\, t_i) dx}{\rho^2\; t_i^2 (k^2 + t_i^2) J_0^2(\rho t_i)} \tag{13}$$

Now using [5], equations (2) through (4), the definition of Bessel function and interchanging the order of summations and integration, we find

$$\int_0^1 \theta^{2\lambda+1} (1-\theta^2)^{\alpha}\, P_{\beta}^{a,b}(1-2\theta^2)\; {}_{p'}\psi_{q'}[K_1(1-\theta^2)]$$

$$.\bar{H}_{P,Q}^{M,N}[K_2(1-\theta^2)]\; \bar{H}_{U,V}^{S,T}[K_3(1-\theta^2)]\; J_{\mu}(\theta\tau)\; d\theta$$

$$= \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \sum_{n''=0}^{\infty} \sum_{h=1}^{S} \sum_{r=0}^{\infty} \frac{K_1^n\, K_3^s (-1)^{r+n''} (-\beta)_{n'}\, \sigma(s)\; \left(\frac{\tau}{2}\right)^{\mu+2n''}}{2\; n\,!\;\; n'\,!\;\;\; n''\,!\;\;\; \beta\,!\;\;\; r\,!\;\; \beta_h}$$

$$.\frac{\prod_{j=1}^{p'} \Gamma(e_j + E_j n)\; \Gamma(1+a+\beta)\; (1+a+b+\beta)_{n'}\; \Gamma\left(\lambda + n' + n'' + \frac{\mu}{2} + 1\right)}{\prod_{j=1}^{q'} \Gamma(f_j + F_j n)\; \Gamma(1+a+n')\; \Gamma(1+\mu+n'')}$$

$$\bar{H}_{P+1,Q+1}^{M,\,N+1}\left[K_2 \left|\begin{matrix}(-\alpha-n-s,1;1);\, (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q}, (-1-\lambda-n-n'-n''-\alpha-s-\frac{\mu}{2},1;1)\end{matrix}\right.\right] \tag{14}$$

where

$$Re(a) \;>\; -1,\; Re(b) \;>\; -1,\; Re(\lambda) \;>\; -1,\; Re(\alpha) \;>\; -1,\; Re(\mu) \;>\; -\frac{1}{2},$$

$$\mathrm{Re}\left(\alpha + \frac{b_j^{'}}{\beta_j^{'}}\right) > 0, \ \mathrm{Re}\left(\alpha + \frac{b_j^{'}}{\beta_j^{'}}\right) > 0, \ (j = 1, ..., Q)$$

Using (14) in view of (6) and (7), we get

$$G_i = \frac{K_0\,\Gamma(1+a+\beta)}{R\,\beta\,!\,(k^2+t_i^2)\,J_0^2(\rho t_i)} \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{n''=0}^{\infty}\sum_{h=1}^{S}\sum_{r=0}^{\infty}$$

$$\cdot\frac{K_1^n\,K_3^s(-1)^{r+n''}(-\beta)_{n'}\,\sigma(s)}{n\ !\ n'\ !\ \ n''\ !\ \ r\ !\ \beta_h}\,\frac{\prod_{j=1}^{p'}\Gamma(e_j+E_jn)\,\Gamma(1+n'+n'')\,(1+a+b+\beta)_{n'}}{\prod_{j=1}^{q'}\Gamma(f_j+F_jn)\,\Gamma(1+a+n')\,\Gamma(1+n'')}$$

$$\bar{H}^{M,\,N+1}_{P+1,Q+1}\left[K_2\,\middle|\begin{matrix}(-\alpha-n-s,1;1),\,(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(-1-n-n'-n''-\alpha-s,1;1)\end{matrix}\right] \tag{15}$$

Now combining the equations (9) and (15), we get

$$w = L_1\sum_i \frac{L_2}{(k^2+t_i^2)}\,[J_0(xt_i) - J_0(\rho t_i)] \tag{16}$$

where

$$L_1 = \frac{K_0\,\Gamma(1+a+\beta)}{R\ \ \beta\,!}$$

and

$$L_2 = \frac{1}{t_i^2\,J_0^2(\rho t_i)}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{n''=0}^{\infty}\sum_{h=1}^{S}\sum_{r=0}^{\infty}\frac{K_1^n\ K_3^s(-1)^{r+n''}(-\beta)_{n'}\,\sigma(s)}{n\ !\ \ n'\ !\ \ n''\ !\ \ r\ !\ \ \beta_h}$$

$$\cdot\frac{\prod_{j=1}^{p'}\Gamma(e_j+E_jn)\,\Gamma(1+n'+n'')\,(1+a+b+\beta)_{n'}}{\prod_{j=1}^{q'}\Gamma(f_j+F_jn)\,\Gamma(1+a+n')\,\Gamma(1+n'')}$$

$$\bar{H}^{M,\,N+1}_{P+1,Q+1}\left[K_2\,\middle|\begin{matrix}(-\alpha-n-s,1;1);\,(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(-1-n-n'-n''-\alpha-s,1;1)\end{matrix}\right]$$

Now the radial displacement y can be obtained by using equation (8) and (9) as

$$y = \frac{k^2t^2x}{24} - \frac{1}{2}\sum_{i=1}^{\infty}G_i^2t_i^2\left[\frac{x}{2}\left\{J_i^{'2}(xt_i) + \left(1 - \frac{1}{x^2t_i^2}\right)J_1^2(xt_i)\right\}\right]$$

$$-\frac{1}{2}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}G_iG_jt_it_j\left[\frac{t_i\,J_2(xt_i)J_1(xt_j) - t_j\,J_2(xt_j)\,J_1(xt_i)}{t_i^2 - t_j^2}\right] + C_1,\ i \neq j \tag{17}$$

where C_1 is the constant of integration.
Applying the boundary condition

$$y = 0 \ \text{ at } x = \rho \text{ and } J_1(\rho t_i) = 0, \text{ we get}$$

$$C_1 = \frac{-k^2t^2\rho}{24} + \frac{1}{4}\sum_{i=1}^{\infty} G_i^2\, t_i^2\, \rho\, J_1'^2(\rho t_i). \tag{18}$$

Hence the radial displacement y is established as

$$y \; = \; \frac{k^2t^2(x-\rho)}{24} - \frac{1}{2}\sum_{i=1}^{\infty} G_i^2 t_i^2 \left[\frac{x}{2}\left\{J_i'^2(xt_i) + \left(1 - \frac{1}{x^2t_i^2}\right) J_1^2(xt_i)\right\}\right]$$

$$-\frac{1}{2}\sum_{i=1}^{\infty}\sum_{\substack{j=1\\ i\neq j}}^{\infty} G_iG_jt_it_j\left[\frac{t_i\, J_2(xt_i)J_1(xt_j) - t_j\, J_2(xt_j)\, J_1(xt_i)}{t_i^2 - t_j^2}\right] + \frac{1}{4}\sum_{i=1}^{\infty} G_i^2\, t_i^2\rho\, J_0^2\,(\rho t_i)$$

3. Applications

(3.A) The deflection given by equation (16) can be used to evaluate the boundary stresses at the surface of the plate which for the circular plate, are given by [1] as

$$\sigma_x = -\frac{6R}{t^2}\left(\frac{d^2w}{dx^2} + \frac{\nu}{x}\frac{dw}{dx}\right) \tag{19}$$

and

$$\sigma_\theta = -\frac{6R}{t^2}\left(\nu\,\frac{d^2w}{dx^2} + \frac{1}{x}\frac{dw}{dx}\right) \tag{20}$$

where ν is the Poisson's ratio.
By using (16), we get

$$\sigma_x = -\frac{6R}{t^2}\, L_1 \sum_i \frac{L_2}{(k^2+t_i^2)}\left[J_0''(xt_i) + \frac{\nu}{x}\, J_0'(x\, t_i)\right] \tag{21}$$

and

$$\sigma_\theta = -\frac{6R}{t^2}\, L_1 \sum_i \frac{L_2}{(k^2+t_i^2)}\left[\nu\, J_0''(xt_i) + \frac{1}{x}\, J_0'(x\, t_i)\right] \tag{22}$$

Now, putting $x = 0$ in (21) and (22), we get the bending stresses at the centre of the plate as

$$(\sigma_x)_{x=0} \; = \; (\sigma_\theta)_{x=0} \; = \; \frac{3R}{t^2}\, L_1 \sum_i \frac{L_2}{(k^2+t_i^2)}(\nu+1)\, t_i^2, \tag{23}$$

Also by putting $x = \rho$, the bending stresses at the edge of the plate are obtained as

$$(\sigma_x)_{x=\rho} = \frac{6RL_1}{t^2}\sum_i \frac{L_2}{(k^2+t_i^2)}\, t_i^2\, J_0(\rho t_i) \tag{24}$$

and

$$(\sigma_\theta)_{x=\rho} = \frac{6RL_1}{t^2}\sum_i \frac{L_2}{(k^2+t_i^2)}\,\nu\, t_i^2\, J_0(\rho t_i) \tag{25}$$

(3.B) When $k = 0$, the differential equation (7) corresponds to that of small deflection equation and then equation (16) leads to

$$w = L_1 \sum_i \frac{L_2}{t_i^2}[J_0(xt_i) - J_0(\rho t_i)] \tag{26}$$

(3.C) By using $x = 0$, we obtain the deflection w_0 at the centre of the plate as

$$w_0 = L_1 \sum_i \frac{L_2}{(k^2 + t_i^2)}[1 - J_0(\rho t_i)] \tag{27}$$

whereas the small deflection will be given by

$$w_0 = L_1 \sum_i \frac{L_2}{t_i^2}[1 - J_0(\rho t_i)] \tag{28}$$

4. Special Cases

(A) By setting $\alpha_j = 1, \beta_j = 1, A_j = 1, B_j = 1, \forall j$ for $\bar{H}^{S,T}_{U,V}\left\{K_3\left(1 - \frac{x^2}{\rho^2}\right)\right\}$ in equation (6) all the results reduce to known result obtained by V.B.L. Chaurasia and R.C. Meghwal [4].

(B) By taking $A_j = B_j = 1$ and $A_j^{'} = B_j^{'} = 1$ for $\bar{H}^{M,N}_{P,Q}$ and $\bar{H}^{S,T}_{U,V}$ in the load p(x), both the $\bar{H}$-functions reduces to the Fox's H-function. Then we obtain the deflection as

$$w = D_1 \sum_i \frac{D_2}{(k^2 + t_i^2)}[J_0(xt_i) - J_0(\rho t_i)] \tag{29}$$

where

$$D_1 = \frac{K_0}{R}\frac{\Gamma(1 + a + \beta)}{\beta\,!} \tag{30}$$

and

$$D_2 = \frac{1}{t_i^2 J_0^2(\rho t_i)} \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{n''=0}^{\infty}\sum_{h=1}^{S}\sum_{r=0}^{\infty} \frac{K_1^n\, K_3^{s'}(-1)^{r+n''}(-\beta)_{n'}}{n\,!\;n'\,!\;n''\,!\;r\,!\;\beta_h}\varphi(s')\left(\frac{\rho t_i}{2}\right)^{2n''}$$

$$\cdot\frac{(1 + a + b + \beta)_{n'}\,\Gamma(1 + n + n'')\prod_{j=1}^{p'}\Gamma(e_j + E_j n)}{\Gamma(1 + a + n')\,\Gamma(1 + n'')\prod_{j=1}^{q'}\Gamma(f_j + F_j n)}$$

$$H^{M,\,N+1}_{P+1,Q+1}\left[K_2\left|\begin{smallmatrix}(-\alpha-n-s',1;1),\,(a_j,\alpha_j)_{1,P}\\ (b_j,\beta_j)_{1,Q},\,(-1-n-n'-n''-\alpha-s',1;1)\end{smallmatrix}\right.\right]$$

whereas we get the small deflection as

$$w = D_1 \sum_i \frac{D_2}{t_i^2}[J_0(xt_i) - J_0(\rho t_i)] \tag{31}$$

In this case, the deflection at the centre of the plate is given by

$$w_0 = D_1 \sum_i \frac{D_2}{(k^2 + t_i^2)}[1 - J_0(\rho t_i)] \tag{32}$$

(C) By replacing $\bar{H}^{S,T}_{U,V}[K_3(1-\theta^2)]$ by
${}_U\bar{\psi}_V \left[\begin{matrix} (a_j',\alpha_j';A_j')_{1,U}; \\ (b_j',\beta_j';B_j')_{1,V}; \end{matrix} k_3(1-\theta^2) \right]$
and

$$\bar{H}^{M,N}_{P,Q}[K_2(1-\theta^2)] \text{ by } {}_P\bar{\psi}_Q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,P}; \\ (b_j,\beta_j;B_j)_{1,Q}; \end{matrix} K_2(1-\theta^2) \right]$$

in equation (6), we obtain the deflection as

$$w = D_1 \sum_i \frac{D_3}{(k^2 + t_i^2)} [J_0(xt_i) - J_0(\rho t_i)] \tag{33}$$

where D_1 is given by (30) and

$$D_3 = \frac{1}{t_i^2 J_0^2(\rho t_i)} \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \sum_{n''=0}^{\infty} \sum_{\ell=0}^{\infty} \frac{K_1^n\, K_3^{\ell} (-1)^{n''} (-\beta)_{n'}}{n!\ n'!\ n''!\ \ell!} \left(\frac{\rho t_i}{2}\right)^{n''}$$

$$\cdot \frac{(1+a+b+\beta)_{n'} \Gamma(1+n+n'') \prod_{j=1}^{p'} \Gamma(e_j + E_j n) \prod_{j=1}^{P} \{\Gamma(e_j' + E_j' \ell)\}^{A_j}}{\Gamma(1+a+n')\,\Gamma(1+n'') \prod_{j=1}^{q'} \Gamma(f_j + F_j n) \prod_{j=1}^{Q} \{\Gamma(f_j' + F_j' \ell)\}^{B_j}}$$

$$\bar{H}^{1,\ P+1}_{P+1,Q+2} \left[(-K_2) \left| \begin{matrix} (-\alpha-n-\ell,1;1),\ \{(1-a_j),\alpha_j;A_j\}_{1,P} \\ (0,1),\ \{(1-b_j),\beta_j;B_j\}_{1,Q},\ (-1-n-n'-n''-\alpha-\ell,1;1) \end{matrix} \right. \right]$$

The small deflection in this case is given by

$$w = D_1 \sum_i \frac{D_3}{t_i^2} [J_0(x\, t_i) - J_0(\rho\, t_i)], \tag{34}$$

also, the deflection at the centre of the plate is,

$$w_0 = D_1 \sum_i \frac{D_3}{(k^2 + t_i^2)} [1 - J_0(\rho t_i)] \tag{35}$$

(D) By replacing $\bar{H}^{S,T}_{U,V}[K_3(1-\theta^2)]$ by $g(S,T,U,V;K_3(1-\theta^2))$ and $\bar{H}^{M,N}_{P,Q}[K_2(1-\theta^2)]$ by $g(M,N,P,Q;\ K_2(1-\theta^2))$ the special cases of $\bar{H}$-function ([10], eqn. (6.10), p.306, [6], p.4119-4128) in equation (6), we obtain the deflection as

$$w = D_1 \sum_i \frac{D_4}{(k^2 + t_i^2)} [J_0(xt_i) - J_0(\rho t_i)], \tag{36}$$

where D_1 is given by (30) and

$$D_4 = \frac{1}{t_i^2 J_0^2(\rho t_i)} \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \sum_{n''=0}^{\infty} \sum_{r=0}^{\infty} \frac{K_1^n\, K_3^r\, K_{d-1}^2 (-\beta)_{n'} (-1)^{n''-Q}}{n!\ n'!\ n''!\ r!}$$

$$\cdot \frac{C_2\, f(r)\, \Gamma(Q+1)\Gamma\left(\frac{1}{2}+\frac{P}{2}\right)}{2^{2+Q}\sqrt{\pi}\,\Gamma(M)\,\Gamma\left(M-\frac{P}{2}\right)} \left(\frac{\rho t_i}{2}\right)^{2n''} \frac{(1+a+b+\beta)_{n'}\,\Gamma(1+n+n'')\prod_{j=1}^{p'}\Gamma(e_j+E_j n)}{\Gamma(1+a+n')\,\Gamma(1+n'')\prod_{j=1}^{q'}\Gamma(f_j+F_j n)}$$

$$\bar{H}_{4,4}^{1,\,4}\left[(-K_2)\left|\begin{matrix}(-\alpha-n-r,1;1),\,(1-M,1,1),\,\left(1-M+\frac{P}{2},1;1\right),(1-N,1;1+Q)\\(0,1),\left(-\frac{P}{2},1;1\right),(-N,1;1+Q),(-1-n-n'-n''-\alpha-r,1;1)\end{matrix}\right.\right],$$

where

$C_2 = \frac{2^{-V-2}\Gamma(V+1)\,B\left(\frac{1}{2},\frac{1}{2}+\frac{U}{2}\right)}{\pi}$ and

$$f(r) = \frac{\left(S-\frac{U}{2}\right)_r (S)_r (T+r)^{-(1+V)}}{\left(1+\frac{U}{2}\right)_r}.$$

The small deflection is given by

$$w = D_1 \sum_i \frac{D_4}{(k^2+t_i^2)}\left[J_0(xt_i) - J_0(\rho t_i)\right] \qquad (37)$$

and the deflection at the centre of the plate is

$$w_0 = D_1 \sum_i \frac{D_4}{(k^2+t_i^2)}\left[1 - J_0(\rho t_i)\right] \qquad (38)$$

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