Journal of Fractional Calculus and Applications
Vol. 4(3S)(5th. Symposium of Fractional Calculus and Applications group)
Aug. 12, 2013, No. 7, pp. 1-11.
ISSN: 2090-5858.
http://www.fcaj.webs.com/

NUMERICAL SOLUTION FOR NONLINEAR QUADRATIC INTEGRAL EQUATIONS

E. A. A. ZIADA

Abstract. We are concerned here with a nonlinear quadratic integral equation (QIE) of Volterra type. The existence of a unique solution will be proved. Convergence analysis of Adomian decomposition method (ADM) applied to these type of equations is discussed and the maximum absolute truncated error of Adomian's series solution is estimated. Sometimes, when ADM is used, it produces difficult integrals so, a numerical implementation technique (NIT) is used to overcome this problem.

1. Introduction

Quadratic integral equations (QIEs) are often applicable in the theory of radiative transfer, kinetic theory of gases, theory of neutron transport and traffic theory. The quadratic integral equations have been studied in several papers (see for examples [1]-[6]). In these papers, the authors discussed only the existence of the solution to QIEs and there is no methods given to solve them, while in paper [7] the authors presented two methods; (ADM and repeated trapezoidal) to solve the QIE

$$x(t) = p(t) \ + \ ax^n(t) \int_0^t k(t,s)x^m(s) \ ds$$

The authors in paper [8] proved the existence and the uniqueness of continuous solution for the QIE

$$x(t) = a(t) \ + \ g(t,x(t)) \int_0^t f(s,x(s)) \ ds$$

with conditions that $g(t,x(t))$ and $f(s,x(s))$ are bounded functions.

In this paper, ADM is used to solve QIE:

$$x\left(t\right) = p\left(t\right) + f\left(t, x\left(t\right)\right) \int_0^t k\left(t,s\right) g(s, x\left(s\right))ds, \tag{1}$$

where $|x(t)| < r_0, |f(t,0)| < b_1, |g(s,0)| < b_2, k\left(t,s\right)$ is continuous in $t \ \forall \ s \in I = [0,T]$ and $|k\left(t,s\right)| < k$, $T \in R^+$, $p\left(t\right) \in C\left(I\right), f\left(t,x\right)$ and $g(s,x)$ are continuous

---

2000 *Mathematics Subject Classification.* 26A33, 65L05, 65L20.

Key words and phrases. Nonlinear quadratic integral equation; Banach space; Fixed point theorem; Convergence analysis; Adomian method; numerical implementation technique.

functions in the two arguments t, s and satisfies Lipschitz condition with Lipschitz constants c_1 and c_2 such as,

$$\begin{aligned} |f(t,x) - f(t,y)| &\leq c_1 |x-y|, \\ |g(s,x) - g(s,y)| &\leq c_2 |x-y|. \end{aligned} \tag{2}$$

This method has many advantages, it is efficiently works with different types of linear and nonlinear equations in deterministic or stochastic fields and gives an analytic solution for all these types of equations without linearization or discretization [9]-[16]. The contribution of this work can be summarized in the following five points:

- Introducing the sufficient condition that guarantees the existence of a unique solution to our problem (see Theorem 1).
- Based on the above point and using Adomian polynomials formula suggested in [17] convergence of ADM is discussed (see Theorem 2).
- Using point two, the maximum absolute truncated error of the Adomian's series solution is estimated (see Theorem 3).
- Applying a numerical implementation technique (NIT) to overcome evaluating difficult integrals, if any, produced from using ADM.
- Preparation of algorithms using MATHEMATICA package to solve the given numerical examples.

2. The Solution Algorithm

The solution algorithm of equation (1) using ADM is,

$$x_0(t) = p(t), \tag{3}$$

$$x_i(t) = A_{(i-1)}(t) \int_0^t k(t,s)\, B_{(i-1)}(s)\, ds. \tag{4}$$

where A_i and B_i are Adomian polynomials of the nonlinear terms $g(t,x)$ and $f(s,x)$ respectively, which have the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[g\left(t, \sum_{i=0}^{\infty} \lambda^i x_i\right)\right]_{\lambda=0},$$

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(t, \sum_{i=0}^{\infty} \lambda^i x_i\right)\right]_{\lambda=0}$$

The solution will be,

$$x(t) = \sum_{i=0}^{\infty} x_i(t) \tag{5}$$

3. Convergence Analysis

3.1. **Existence and Uniqueness theorem.** Define the mapping $F : E \to E$ where E is the Banach space $(C(I), \|\cdot\|)$, the space of all continuous functions on I with the norm $\|x\| = \max_{t \in I} |x(t)|$.

Theorem 1: Let $f(t,x)$ and $g(s,x)$ satisfies the Lipschitz condition (2). If

$$T < \frac{1}{k\,(2c_1c_2r_0 + c_1b_2 + c_2b_1)}$$

then the QIE (1) has a unique solution $x \in C(I)$.

Proof: The mapping $F : E \to E$ is defined as,

$$Fx = p(t) + f(t, x(t)) \int_0^t k(t, s)\, g(s, x(s))ds,$$

Let $x, y \in E$, then

$$\begin{aligned}
Fx - Fy &= f(t, x(t)) \int_0^t k(t, s)\, g(s, x(s))ds - f(t, y(t)) \int_0^t k(t, s)\, g(s, y(s))ds \\
&= f(t, x(t)) \int_0^t k(t, s)\, g(s, x(s))ds - f(t, y(t)) \int_0^t k(t, s)\, g(s, x(s))ds \\
&\quad + f(t, y(t)) \int_0^t k(t, s)\, g(s, x(s))ds - f(t, y(t)) \int_0^t k(t, s)\, g(s, y(s))ds \\
&= [f(t, x(t)) - f(t, y(t))] \int_0^t k(t, s)\, g(s, x(s))ds \\
&\quad + f(t, y(t)) \int_0^t k(t, s)\, [g(s, x(s)) - g(s, y(s))]\, ds
\end{aligned}$$

which implies that

$$\begin{aligned}
\|Fx - Fy\| &\leq \max_{t\in I} \left| f(t, x(t)) - f(t, y(t)) \int_0^t k(t, s)\, g(s, x(s))ds \right| \\
&\quad + \max_{t\in I} \left| f(t, y(t)) \int_0^t k(t, s)\, [g(s, x(s)) - g(s, y(s))]\, ds \right| \\
&\leq \max_{t\in I} |f(t, x(t)) - f(t, y(t))| \int_0^t |k(t, s)|\, |g(s, x(s))|\, ds \\
&\quad + \max_{t\in I} |f(t, y(t))| \int_0^t |k(t, s)|\, |g(s, x(s)) - g(s, y(s))|\, ds \\
&\leq c_1 k \max_{t\in I} |x(t) - y(t)| \int_0^t |g(s, x(s)) - g(s, 0)| + |g(s, 0)|\, ds \\
&\quad + c_2 k \max_{t\in I} |x(t) - y(t)|\, [|f(t, y(t)) - f(t, 0)| + |f(t, 0)|] \int_0^t ds \\
&\leq c_1 k\, [c_2\, |x(t)| + b_2]\, T\, \|x - y\| + c_2 k\, [c_1\, |y(t)| + b_1]\, T\, \|x - y\| \\
&\leq [c_1 c_2 r_0 + c_1 b_2]\, kT\, \|x - y\| + [c_1 c_2 r_0 + c_2 b_1]\, kT\, \|x - y\| \\
&\leq (2c_1 c_2 r_0 + c_1 b_2 + c_2 b_1)\, kT\, \|x - y\| \\
&\leq \alpha\, \|x - y\|
\end{aligned}$$

under the condition $0 < \alpha < 1$, the mapping F is contraction and hence for $T < \frac{1}{k(2c_1c_2r_0+c_1b_2+c_2b_1)}$ there exists a unique solution $x \in C(I)$ of the problem (1) and this completes the proof. ■

3.2. **Proof of convergence.** **Theorem 2**: *Let the solution of the QIE (1) exist. If $|x_1(t)| < l, l$ is a positive constant, then the series solution (5) of the QIE (1) using ADM converges.*

Proof: Define the sequence $\{S_p\}$ such that, $S_p = \sum_{i=0}^{p} x_i(t)$ is the sequence of partial sums from the series solution $\sum_{i=0}^{\infty} x_i(t)$, and we have

$$\begin{aligned} f(t,x) &= \sum_{i=0}^{\infty} A_i, \\ g(t,x) &= \sum_{i=0}^{\infty} B_i. \end{aligned}$$

Let S_p and S_q be two arbitrary partial sums with $p > q$. Now, we are going to prove that $\{S_p\}$ is a Cauchy sequence in this Banach space E.

$$\begin{aligned} S_p - S_q &= \sum_{i=0}^{p} x_i - \sum_{i=0}^{q} x_i \\ &= \sum_{i=0}^{p} A_{(i-1)}(t) \int_0^t k(t,s) \sum_{i=0}^{p} B_{(i-1)}(s)\, ds - \sum_{i=0}^{q} A_{(i-1)}(t) \int_0^t k(t,s) \sum_{i=0}^{q} B_{(i-1)}(s)\, ds \\ &= \sum_{i=0}^{p} A_{(i-1)}(t) \int_0^t k(t,s) \sum_{i=0}^{p} B_{(i-1)}(s)\, ds - \sum_{i=0}^{q} A_{(i-1)}(t) \int_0^t k(t,s) \sum_{i=0}^{p} B_{(i-1)}(s)\, ds \\ &\quad + \sum_{i=0}^{q} A_{(i-1)}(t) \int_0^t k(t,s) \sum_{i=0}^{p} B_{(i-1)}(s)\, ds - \sum_{i=0}^{q} A_{(i-1)}(t) \int_0^t k(t,s) \sum_{i=0}^{q} B_{(i-1)}(s)\, ds \\ &= \left[\sum_{i=0}^{p} A_{(i-1)}(t) - \sum_{i=0}^{q} A_{(i-1)}(t)\right] \int_0^t k(t,s) \sum_{i=0}^{p} B_{(i-1)}(s)\, ds \\ &\quad + \sum_{i=0}^{q} A_{(i-1)}(t) \int_0^t k(t,s) \left[\sum_{i=0}^{p} B_{(i-1)}(s) - \sum_{i=0}^{q} B_{(i-1)}(s)\right] ds \\ \|S_p - S_q\| &\leq \max_{t\in I} \left| \sum_{i=q+1}^{p} A_{(i-1)}(t) \int_0^t k(t,s) \sum_{i=0}^{p} B_{(i-1)}(s)\, ds \right| \\ &\quad + \max_{t\in I} \left| \sum_{i=0}^{q} A_{(i-1)}(t) \int_0^t k(t,s) \sum_{i=q+1}^{p} B_{(i-1)}(s)\, ds \right| \\ &\leq \max_{t\in I} \left| \sum_{i=q}^{p-1} A_i(t) \right| \int_0^t |k(t,s)| \left| \sum_{i=0}^{p} B_{(i-1)}(s)\, ds \right| ds \\ &\quad + \max_{t\in I} \left| \sum_{i=0}^{q} A_{(i-1)}(t) \right| \int_0^t |k(t,s)| \left| \sum_{i=q}^{p-1} B_i(s) \right| ds \\ &\leq k \max_{t\in I} |f(t,S_{p-1}) - f(t,S_{q-1})| \int_0^t |g(t,S_p)|\, ds \\ &\quad + k \max_{t\in I} |f(t,S_q)| \int_0^t |g(t,S_{p-1}) - g(t,S_{q-1})|\, ds \\ &\leq c_1 kT [c_2 r_0 + b_2] \max_{t\in I} |S_{p-1} - S_{q-1}| + c_2 kT [c_1 r_0 + b_1] \max_{t\in I} |S_{p-1} - S_{q-1}| \\ &\leq \alpha \|S_{p-1} - S_{q-1}\| \end{aligned}$$

Let $p = q + 1$ then,

$$\|S_{q+1} - S_q\| \leq \alpha \|S_q - S_{q-1}\| \leq \alpha^2 \|S_{q-1} - S_{q-2}\| \leq \cdots \leq \alpha^q \|S_1 - S_0\|$$

From the triangle inequality we have,

$$\begin{aligned}
\|S_p - S_q\| &\leq \|S_{q+1} - S_q\| + \|S_{q+2} - S_{q+1}\| + \cdots + \|S_p - S_{p-1}\| \\
&\leq \left[\alpha^q + \alpha^{q+1} + \cdots + \alpha^{p-1}\right] \|S_1 - S_0\| \\
&\leq \alpha^q \left[1 + \alpha + \cdots + \alpha^{p-q-1}\right] \|S_1 - S_0\| \\
&\leq \alpha^q \left[\frac{1 - \alpha^{p-q}}{1 - \alpha}\right] \|x_1(t)\|
\end{aligned}$$

Now $0 < \alpha < 1$, and $p > q$ implies that $(1 - \alpha^{p-q}) \leq 1$. Consequently,

$$\begin{aligned}
\|S_p - S_q\| &\leq \frac{\alpha^q}{1 - \alpha} \|x_1(t)\| \\
&\leq \frac{\alpha^q}{1 - \alpha} \max_{t \in I} |x_1(t)|
\end{aligned}$$

but, $|x_1(t)| < l$ and as $q \to \infty$ then, $\|S_p - S_q\| \to 0$ and hence, $\{S_p\}$ is a Cauchy sequence in this Banach space E and the series $\sum_{i=0}^{\infty} x_i(t)$ converges. ∎

3.3. **Error analysis.** **<u>Theorem 3</u>**: *The maximum absolute truncation error of the series solution (5) to the problem (1) is estimated to be,*

$$\max_{t \in I} \left| x(t) - \sum_{i=0}^{q} x_i(t) \right| \leq \frac{\alpha^q}{1 - \alpha} \max_{t \in I} |x_1(t)|$$

<u>Proof:</u> From Theorem 2 we have,

$$\|S_p - S_q\| \leq \frac{\alpha^q}{1 - \alpha} \max_{t \in I} |x_1(t)|$$

but, $S_p = \sum_{i=0}^{p} x_i(t)$ as $p \to \infty$ then, $S_p \to x(t)$ so,

$$\|x(t) - S_q\| \leq \frac{\alpha^q}{1 - \alpha} \max_{t \in I} |x_1(t)|$$

so, the maximum absolute truncation error in the interval I is,

$$\max_{t \in I} \left| x(t) - \sum_{i=0}^{q} x_i(t) \right| \leq \frac{\alpha^q}{1 - \alpha} \max_{t \in I} |x_1(t)|$$

and this completes the proof. ∎

4. Numerical Example

Example 1. *Consider the following nonlinear QIE,*

$$x(t) = \left(t^2 - \frac{t^{13}}{240}\right) + \frac{1}{30} x^2(t) \int_0^t (ts)\, x^3(s)\, ds, \tag{6}$$

which has the exact solution $x(t) = t^2$.

Applying ADM to equation (6), we get

$$x_0(t) = \left(t^2 - \frac{t^{13}}{240}\right),$$
$$x_i(t) = \frac{1}{30} A_{i-1}(t) \int_0^t (ts)\, B_{i-1}(s)\, ds, \quad i \geq 1.$$

where A_i and B_i are Adomian polynomials of the nonlinear terms x^2 and x^3 respectively, and the solution will be,

$$x(t) = \sum_{i=0}^{q} x_i(t)$$

this series solution converges if $T < 1.1649931$. Table 1 shows the absolute error of ADM series solution (when $q = 5$), while table 2 shows the maximum absolute truncated error (using Theorem 3) at different values of q (at $t = 1$).

Table 1: Absolute Error

t	$\lvert x_{Exact} - x_{ADM} \rvert$
0.1	5.65022×10^{-29}
0.2	9.50462×10^{-22}
0.3	1.60002×10^{-17}
0.4	1.59461×10^{-14}
0.5	3.37671×10^{-12}
0.6	2.6843×10^{-10}
0.7	1.08516×10^{-8}
0.8	2.67331×10^{-7}
0.9	4.50658×10^{-6}
1	0.000056163

Table 2 : Max. Absolute Error

q	Max. error
5	0.0000701496
10	7.18332×10^{-7}
15	7.35572×10^{-9}
20	7.53225×10^{-11}

5. Numerical Implementation of ADM

This is a modification to ADM, it is used when the evaluation of the integrals; which appear due to the calculations of the series solution terms, are difficult or impossible analytically. This technique was discussed before by Babolian in [18], he applied it to linear Volterra integral equations of the second kind. Now, we apply this modification for solving QIEs.

5.1. **Description of the numerical implementation technique (NIT).** Applying the classical ADM to equation (1) we get,

$$x_0(t) = p(t), \tag{7}$$
$$x_{n+1}(t) = f(t, x_n(t)) \int_0^t k(t,s)\, g(s, x_n(s)) ds. \tag{8}$$

Now, we will use the numerical method; which given in [19], to approximate the integral term in equation (8). We choose a regular mesh in t, thus setting $t = t_i = ih$ where $h = \frac{1}{n}$ is the fixed step length. Therefore, the integral in (8) can be approximated as,

$$\int_0^{t_i} k(t_i, s)\, g(s, x_n(s))ds \simeq h \sum_{j=0}^{i} w_{ij} k(t_i, s_j)\, g(s_j, x_n(s_j)), \tag{9}$$

where $t_i = s_i$, $i = 0, 1, \ldots, n$. This leads to the following set of nonlinear equations,

$$\begin{aligned} x_{n+1}(t_i) &\simeq (f(t_i, x_n(t_i))) \left(h \sum_{j=0}^{i} w_{ij}\ k(t_i, t_j)\, g(t_j, x_n(t_j)) \right), \\ i &= 0, 1, \ldots \quad n = 0, 1, 2, \ldots. \end{aligned} \tag{10}$$

For choosing suitable weights w_{ij}, we note that for each i the set w_{ij}, $j = 0, 1, \ldots, i$ represents the weights for an $(i+1)$-points quadrature rules of Newton Cotes type for the interval $[0, ih]$. We implement the above idea on the following examples with $n = 20$ or $h = \frac{1}{20}$.

5.2. **Numerical examples.**

Example 2. *Consider the following nonlinear QIE,*

$$x(t) = \left(t - \frac{t^2}{10}(-1 - t + e^t) \right) + \frac{1}{10} x^2(t) \int_0^t (t - s)\, e^{x(s)} ds, \tag{11}$$

and has the exact solution $x(t) = t$.

Applying the standard ADM, we have the following solution algorithm,

$$\begin{aligned} x_0(t) &= \left(t - \frac{t^2}{10}(-1 - t + e^t) \right), \\ x_{n+1}(t) &= \frac{1}{10} x_n^2(t) \int_0^t (t - s)\, e^{x_n(s)} ds, \quad i \geq 1. \end{aligned}$$

Using NIT we get,

$$x_0(t_i) = \left(t_i - \frac{t_i^2}{10}(-1 - t_i + e^{t_i}) \right),$$

$$x_{n+1}(t_i) \simeq \frac{h}{10} x_n^2(t_i) \sum_{j=0}^{i} w_{ij}(t_i - t_j) e^{x_n(t_j)}, \quad i, n = 0, 1, \ldots, 20.$$

Table 3 shows the absolute error of NIT solution, while table 4 shows the absolute error between ADM and NIT solutions. Figure 1 shows NIT and exact solutions.

Table 3 : Absolute Error

t	$\|x_{Exact} - x_{NIT}\|$
0.1	4.32715×10^{-8}
0.2	4.24534×10^{-7}
0.3	2.56867×10^{-6}
0.4	0.0000138591
0.5	0.0000618267
0.6	0.00022579
0.7	0.000697233
0.8	0.00188364
0.9	0.00457387
1	0.0101856

Table 4 : Absolute Error

t	$\|x_{ADM} - x_{NIT}\|$
0.1	1.27579×10^{-7}
0.2	5.17997×10^{-6}
0.3	0.000041088
0.4	0.000175105
0.5	0.000531735
0.6	0.00129887
0.7	0.00271789
0.8	0.00505252
0.9	0.00853627
1	0.0133129

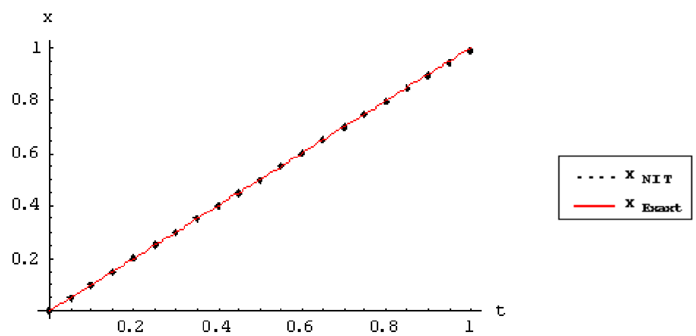

Fig (1): NIT and exact solutions.

Example 3. *Consider the following nonlinear QIE* [1],

$$x(t) = e^{-t} + x(t) \int_0^t \frac{t^2 \ln(1 + s\,|x(s)|)}{2e^{(t+s)}} ds, \quad 0 < t \leq 2. \tag{12}$$

Applying the standard ADM, we have the following solution algorithm,

$$\begin{aligned} x_0(t) &= e^{-t}, \\ x_{n+1}(t) &= x_n(t) \int_0^t \frac{t^2 \ln(1 + s\,|x_n(s)|)}{2e^{(t+s)}} ds, \quad i \geq 1. \end{aligned}$$

Using NIT we get,

$$\begin{aligned} x_0(t_i) &= e^{-t_i}, \\ x_{n+1}(t_i) &\simeq h x_n(t_i) \sum_{j=0}^{i} w_{ij} \frac{t_i^2 \ln(1 + t_j\,|x_n(t_j)|)}{2e^{(t_i+t_j)}}, \qquad i, n = 0, 1, \ldots, 20. \end{aligned}$$

Table 5 the absolute error between ADM and NIT solutions. Figure 2 shows NIT and ADM solutions.

Table 5 : Absolute Error

t	$\lvert x_{ADM} - x_{NIT} \rvert$
0.2	1.8511×10^{-6}
0.4	7.08005×10^{-6}
0.6	0.0000119413
0.8	0.0000143307
1	0.0000138223
1.2	0.0000110818
1.4	7.36944×10^{-6}
1.6	3.87702×10^{-6}
1.8	1.28603×10^{-6}
2	2.6225×10^{-7}

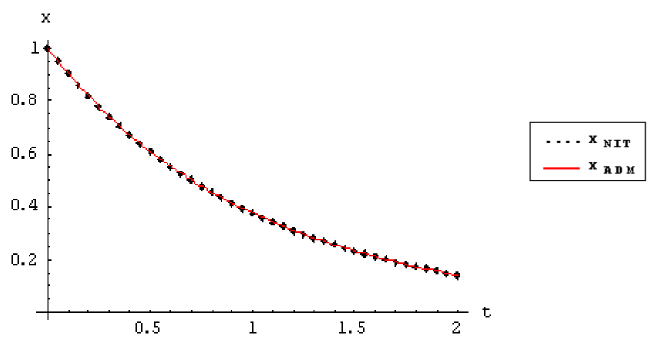

Fig (2): NIT and ADM solutions.

Example 4. *Consider the following nonlinear QIE* [6],

$$x(t) = \frac{t}{e} + \frac{1}{4} x(t) \int_0^t s\left(t + \ln\left(1 + |x(s)|\right)\right) ds, \quad 0 < t \leq 1. \tag{13}$$

Applying the standard ADM, we have the following solution algorithm,

$$\begin{aligned} x_0(t) &= \frac{t}{e}, \\ x_{n+1}(t) &= \frac{1}{4} x_n(t) \int_0^t s\left(t + \ln\left(1 + |x_n(s)|\right)\right) ds, \quad i \geq 1. \end{aligned}$$

Using NIT we get,

$$\begin{aligned} x_0(t_i) &= \frac{t_i}{e}, \\ x_{n+1}(t_i) &\simeq \frac{h}{4} x_n(t_i) \sum_{j=0}^{i} w_{ij}\ t_j \left(t_i + \ln\left(1 + |x_n(t_j)|\right)\right), \quad i, n = 0, 1, \ldots, 20. \end{aligned}$$

Table 6 the absolute error between ADM and NIT solutions. Figure 3 shows NIT and ADM solutions.

Table 6 : Absolute Error

t	$\lvert x_{ADM} - x_{NIT} \rvert$
0.1	1.37936×10^{-7}
0.2	6.26457×10^{-7}
0.3	2.73652×10^{-6}
0.4	0.0000137692
0.5	0.0000593958
0.6	0.000208258
0.7	0.000615591
0.8	0.00159669
0.9	0.00374892
1	0.00815617

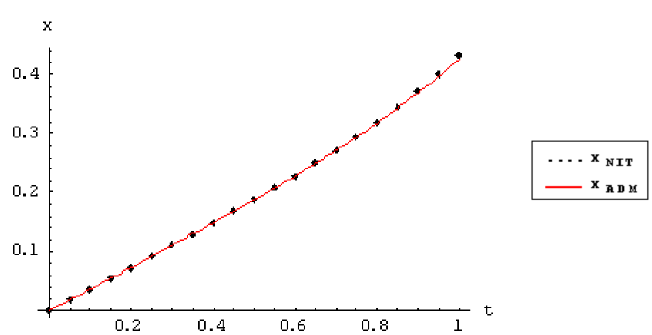

Fig (3): NIT and ADM solutions.

Example 5. *Consider the following nonlinear QIE* [1],

$$x(t) = t^2 + x(t)\int_0^t \frac{(t+s)\,x(s)}{1+x^2(s)}ds, \quad 0 < t \leq 1. \tag{14}$$

Applying the standard ADM, we have the following solution algorithm,

$$\begin{aligned} x_0(t) &= t^2, \\ x_{n+1}(t) &= x_n(t)\int_0^t \frac{(t+s)\,x_n(s)}{1+x_n^2(s)}ds, \quad i \geq 1. \end{aligned}$$

Using NIT we get,

$$\begin{aligned} x_0(t_i) &= t_i^2, \\ x_{n+1}(t_i) &\simeq h x_n(t_i)\sum_{j=0}^{i} w_{ij}\,\frac{(t_i+t_j)\,x_n(t_j)}{1+x_n^2(t_j)}, \qquad i,n = 0,1,\ldots,20. \end{aligned}$$

Figure 4 shows NIT and ADM solutions.

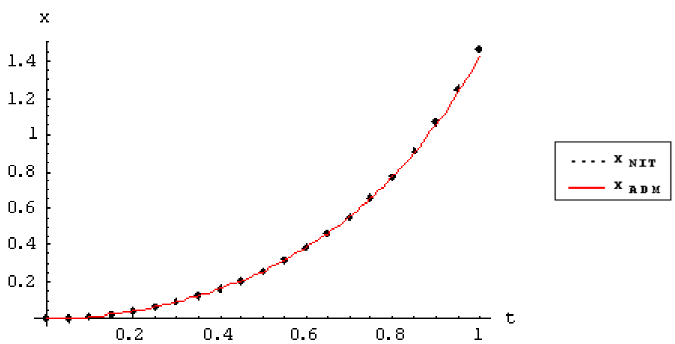

Fig (4): NIT and ADM solutions.

Example 6. *Consider the following nonlinear QIE* [6],

$$x(t) = t^3 + \left(\frac{1}{4}x(t) + \frac{1}{4}\right)\left(\int_0^t t + \cos\left(\frac{x(s)}{1+x^2(s)}\right)\right)ds, \tag{15}$$

Applying the standard ADM, we have the following solution algorithm,

$$\begin{aligned} x_0(t) &= t^3, \\ x_{n+1}(t) &= \left(\frac{1}{4}x_n(t) + \frac{1}{4}\right)\left(\int_0^t t + \cos\left(\frac{x_n(s)}{1+x_n^2(s)}\right)\right)ds, \quad i \geq 1. \end{aligned}$$

Using NIT we get,

$$\begin{aligned} x_0(t_i) &= t_i^3, \\ x_{n+1}(t_i) &\simeq h\left(\frac{1}{4}x_n(t_i) + \frac{1}{4}\right)\sum_{j=0}^{i} w_{ij}\,\left(t_i + \cos\left(\frac{x_n(t_j)}{1+x_n^2(t_j)}\right)\right), \\ i,n &= 0,1,\ldots,20. \end{aligned}$$

Figure 5 shows NIT solution.

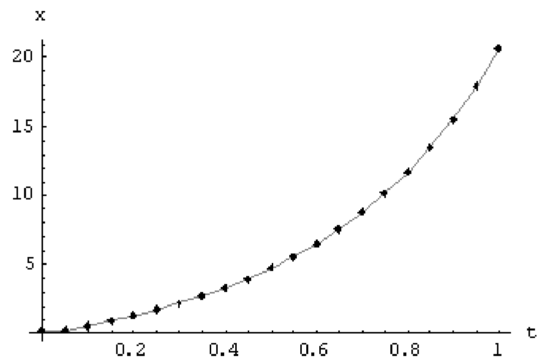

Fig (5): NIT solution.

References

[1] J. Banaś, and A. Martinon, Monotonic solutions of a quadratic integral equation of Volterra type, Comput. Math. Appl. 47 (2004) 271–279.
[2] J. Banaś, M. Lecko, and W.G. El-Sayed, Existence theorems of some quadratic integral equations, J. Math. Anal. Appl. 222 (1998) 276–285.
[3] S. Chandrasekher, Radiative Transfer, Dover, New York, 1960.
[4] S. Hu, M. Khavani, and W. Zhuang, Integral equations arising in the kinetic theory of gases, Appl. Anal. 34 (1989) 261–266.
[5] J. Banaś, J. R. Martinb, and K. Sadaranganib, On solutions of a quadratic integral equation of Hammerstein type, Math. and Comp. Modelling 43 (2006) 97–104.
[6] J. Banaś, J. Caballero, J. Rocha and K. Sadarangani, Monotonic solutions of a slass of quadratic integral equations of Volterra type, Computers and Mathematics with Applications, (2005) 943-952.
[7] A. M. A. El-Sayed, M. M. Saleh, E. A. A. Ziada, Numerical and Analytic Solution for Nonlinear Quadratic Integral Equations, MATH. SCI. RES. J. 12 (8) (2008) 183-191.
[8] A. M. A. El-Sayed, H. H. G. Hashem, Ziada E. A. A., Picard and Adomian Methods for quadratic integral equation, Computational & Applied Mathematics 29 (3) (2010) 2576-2580.
[9] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer, 1995.
[10] G. Adomian, R. Rach, and R. Mayer, Modified decomposition, J. Appl. Math. Comput. 23 (1992) 17-23.
[11] G. Adomian, Stochastic System, Academic press, 1983.
[12] G. Adomian, Nonlinear Stochastic Operator Equations, Academic press, San Diego, 1986.
[13] G. Adomian, Nonlinear Stochastic Systems: Theory and Applications to Physics, Kluwer, 1989.
[14] Y. Cherruault, Convergence of Adomian method, Kybernetes, 18 (1989) 31-38.
[15] K. Abbaoui, and Y. Cherruault, Convergence of Adomian's method Applied to Differential Equations, Computers Math. Applic. 28 (1994) 103-109.
[16] Y. Cherruault, G. Adomian, K. Abbaoui, and R. Rach, Further remarks on convergence of decomposition method, Int. J. of Bio-Medical Computing. 38 (1995) 89-93.
[17] I. L. El-kalla, Convergence of Adomian's method applied to a class of Volterra type integro-differential equations, Int. J. of Differential Equations and Applications 10 (2005) 225-234.
[18] E. Babolian, and A. Davari, Numerical implementation of Adomian decomposition method for linear Volterra integral equations of the second kind, J. Appl. Math. Comput. 165, (2005) 223-227.
[19] L. M. Delves, and J. L. Mohamad, Computational Methods for Integral Equations, Cambridge University Press, 1985.

E. A. A. Ziada, Faculty of Engineering, Delta University for Science and Technology, Gamasa, Egypt.
E-mail address: eng_emanziada@yahoo.com