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GENERALIZED TIME-FRACTIONAL TELEGRAPH EQUATION ANALYTICAL SOLUTION BY SUMUDU AND FOURIER TRANSFORMS

R. S. DUBEY, P. GOSWAMI AND F. B. M. BELGACEM

ABSTRACT. We derive and discuss the analytical solution for the generalized time-fractional telegraph equation with the help of the Sumudu and Fourier transforms. In the process we use Green functions to derive the solution of the said differential equation.

1. Introduction

Fractional differential equations have earned a steadily increasing interest focus due to their amenability in modeling many biological, chemical and physical processes. Indeed, applications related to such equations have attracted considerable attention by scholars in various fields such as acoustics, electro-chemistry, electro-magnetics, and visco-elasticity. By incorporating fractional derivatives into them, many standard differential equations and system, in the time, space or both variables, earned the extra designation of fractional or generalized. The telegraph equation is no exception. The time-fractional telegraph equation dealing with well processes and presenting a study pertaining to asymptotic by making use of the Riemann-Liouville approaches.

The problem we treat in this paper was in part treated by Haubold et al. [3] and Saxena et al. ([10], [11]) but using different definitions for the fractional derivative, since we are using Hifler's. Furthermore, our approach is novel due to the use of the Sumudu transform, only two decades old (see Belgacem et al. [2]). The esteemed reader can see from equation (2.9) below, that the Sumudu of the Hilfer fractional derivative, plays a fundamental role in our scheme towards finding the analytical solution.

In the first part of this paper, we derive the exact solution of the generalized Time-Fractional Telegraph Equation (TFTE). To reach the solution, under integral and series forms, in terms of the Fox H-functions, we use a hybrid method, adjoining Sumudu and Fourier transforms. In the second section, we discuss and derive

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the analytical solution for two basic problems of the TFTE, endowed with the generalized Riemann-Liouville fractional operator $D_{a\pm}^{\alpha,\beta}$. The Sumudu-Fourier hybrid method is then applied to the Cauchy and Signaling problems. The appropriate structure and properties for their Green functions are also provided.

2. Definitions and prerequisites

Fractional calculus theory is an extension of the ordinary calculus theory, defining a generalization of the ordinary differentiation and integration to arbitrary order. Differentiation and integration of fractional order are traditionally defined by the right-sided Riemann-Liouville fractional integral operator I_{a+}^{μ}, the left-sided Riemann-Liouville fractional integral operator I_{a-}^{μ}, and the corresponding Riemann-Liouville fractional derivative operators D_{a+}^{μ} and D_{a+}^{μ} , as follows

$$(I_{a+}^{\mu}f)(x) = \frac{1}{\Gamma(\mu)} \int_{a}^{x} \frac{f(t)}{(x-t)^{1-\mu}}dt, (x>0; Re(\mu) > 0), \tag{2.1}$$

$$(I_{a-}^{\mu}f)(x) = \frac{1}{\Gamma(\mu)} \int_{a}^{x} \frac{f(t)}{(t-x)^{1-\mu}}dt, (x<0; Re(\mu) > 0), \tag{2.2}$$

$$(D_{a\pm}^{\mu}f)(x) = \left(\pm\frac{d}{dx}\right)^{n} (I_{a\pm}^{n-\mu}f)(x), (Re(\mu) \geq 0; n = [Re(\mu)]+1), \tag{2.3}$$

where function, f , is locally integrable, Re(μ) represents the real part of the complex number $\mu \in$ C and [Re(μ)] denotes the greatest integer in Re(μ).
Generalized Riemann-Liouville fractional derivative of order α $(0 < \alpha < 1)$ and type β $(0 \leq \beta \leq 1)$ was recently introduced ([4]-[8] and [9]). The right sided and left sided fractional derivative $D_{a+}^{\alpha,\beta}$ *and* $D_{a-}^{\alpha,\beta}$ respectively of order α $(0 < \alpha < 1)$ and type β $(0 \leq \beta \leq 1)$ with respect to x is defined as

$$(D_{a\pm}^{\alpha,\beta}f)\ (x) \ = \ \left(\pm I_{a\pm}^{\beta(1-\alpha)}\frac{d}{dx}\,(I_{a\pm}^{(1-\beta)(1-\alpha)}f)\right)(x), \tag{2.4}$$

whenever right hand side of (2.4) exists. When we consider $\beta = 0$, it provides Classical Riemann-Liouville fractional derivative operator and similarly when $\beta =$ 1, we obtain fractional derivative operator introduced by Liouville. It is known as Hilfer fractional derivative operator also and its applications were studied by several authors like R. Hilfer [8].
With the help of (2.1),(2.2) and (2.3), taking n=1, the operator $D_{a\pm}^{\alpha,\beta}$ can be written as

$$(D_{a\pm}^{\alpha,\beta}f)\ (x) \ = \ [\pm\ I_{a\pm}^{\beta(1-\alpha)}(D_{a\pm}^{\alpha+\beta-\alpha\beta}f)]\ (x). \tag{2.5}$$

We have the definition of Fourier transform

$$F\left\{\frac{\partial^{\alpha}}{\partial x^{\alpha}}f\,(x,t)\right\}(k) \ = \ (-ik)^{\alpha}\ F\,[f\,(x,t)]\,(k). \tag{2.6}$$

Watugala [12] introduced a new integral transform named the Sumudu transform in early 90's and applied it to the solution of ordinary differential equation in control engineering problems. The Sumudu transform is defined over the set of functions:

$$A = \left\{f(t)\,\middle|\,\exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{|\,t\,|\tau_j},\ if\ t \in (-1)^j \times [0,\infty)\right\}, \tag{2.7}$$

the Sumudu transform is defined by

$$\tilde{G}(u) = S[f(t)] = \int_{0}^{\infty} f(ut)e^{-t}dt, \quad u \in (-\tau_1, \tau_2) \tag{2.8}$$

for other details see ([2], [?], [?]) etc. Sumudu transform of Hilfer fractional derivative operator is given as,

$$S\left[(D_{0+}^{\alpha,\beta}f)(x)\right](u) = u^{-\alpha+1}S\left[f(x)\right](u) - u^{-\beta(\alpha-1)}[I_{0+}^{(1-\beta)(1-\alpha)}f](0+)\ (0<\alpha<1), \tag{2.9}$$

where $(I_{0+}^{(1-\beta)(1-\alpha)}f)(0+)$ is the Riemann-Liouville fractional integral of order $(1-\beta)(1-\alpha)$ evaluated in the limit as t $\to 0^+$.

3. Main results

Fractional differential equations (FDEs) have been always a point of great amount of interest due to its approaches in various fields and their requirements in modeling of system under consideration provided by fractional derivatives for example, in modeling of mathematical biology and in modeling of many chemical processed etc.

We consider the following time-fractional telegraph equation (TFTE)

$$D_{0+}^{2\alpha,\beta}U(x,t) + 2a\, D_{0+}^{\alpha,\beta}U(x,t) = d\,\frac{\partial^2}{\partial x^2}U(x,t) + f(x,t),\ t \in R^+ \tag{3.1}$$

where $0 < \alpha \leq \frac{1}{2}$ *and* $0 \leq \beta \leq 1$. For the TFTE (3.1), we will consider two basic problems.

Problem 1. TFTE in a whole-space domain (Generalized Cauchy Problem)

$$I_{0+}^{(1-\beta)(1-2\alpha)}U(x,0) = \varphi_1(x)\ ;\ I_{0+}^{(1-\beta)(1-\alpha)}U(x,0) = \varphi_2(x);$$
$$U(\mp\infty,t) = 0,\ t > 0,\ x \in R. \tag{3.2}$$

Problem 2. TFTE in a half-space domain (Generalized Signaling Problem)

$$I_{0+}^{(1-\beta)(1-2\alpha)}U(x,0) = 0,\ I_{0+}^{(1-\beta)(1-\alpha)}U(x,0) = 0\ ;$$
$$U(\mp\infty,t) = 0\ ;\ U(0,t) = g(t),\ t > 0,\ x \in R. \tag{3.3}$$

Theorem 3.1. *The solution of the Generalized Cauchy Problem,*

$$D_{0+}^{2\alpha,\beta}U(x,t) + 2a\, D_{0+}^{\alpha,\beta}U(x,t) = d\,\frac{\partial^2}{\partial x^2}U(x,t) + f(x,t), \tag{3.4}$$

where $0 < \alpha \leq \frac{1}{2}$ *and* $0 \leq \beta \leq 1$, *with initial condition*

$$I_{0+}^{(1-\beta)(1-2\alpha)}U(x,0) = \varphi_1(x)\ ;\ I_{0+}^{(1-\beta)(1-\alpha)}U(x,0) = \varphi_2(x);$$
$$U(\mp\infty,t) = 0,\ t > 0,\ x \in R. \tag{3.5}$$

is given in the following proof.

Proof. Applying Sumudu transform of equation (3.4), we find

$$u^{-2\alpha}\tilde{U}(x,u) - u^{-\beta(2\alpha-1)-1}\varphi_1(x) + 2au^{-\alpha}\tilde{U}(x,u) - 2a\,u^{-\beta(\alpha-1)-1}\varphi_2(x)$$
$$= d\,\frac{\partial^2}{\partial x^2}\tilde{U}(x,u) + \tilde{f}(x,u). \tag{3.6}$$

Taking spatial Fourier transform on both side of the above equation (3.6), we get

$$u^{-2\alpha}\tilde{\tilde{U}}(k,u) - u^{-\beta(2\alpha-1)-1}\tilde{\varphi}_1(k) + 2au^{-\alpha}\tilde{\tilde{U}}(k,u) - 2au^{-\beta(\alpha-1)-1}\tilde{\varphi}_2(k) = -dk^2\tilde{\tilde{U}}(k,u) + \tilde{\tilde{f}}(k,u),$$

or

$$(u^{-2\alpha}+2au^{-\alpha}+dk^2)\tilde{\bar{U}}(k,u)=u^{-\beta(2\alpha-1)-1}\tilde{\varphi}_1(k)+2au^{-\beta(\alpha-1)-1}\tilde{\varphi}_2(k)+\tilde{\bar{f}}(k,u), \tag{3.7}$$

we obtain

$$\begin{aligned}\tilde{\bar{U}}(k,u) &= \frac{u^{-\beta(2\alpha-1)-1}\tilde{\varphi}_1(k)+2a\,u^{-\beta(\alpha-1)-1}\tilde{\varphi}_2(k)}{(u^{-2\alpha}+2au^{-\alpha}+dk^2)}+\frac{1}{(u^{-2\alpha}+2au^{-\alpha}+dk^2)}\,\tilde{\bar{f}}(k,u)\\ &= \tilde{\bar{G}}_1(k,u)\,\tilde{\varphi}_1(k)\;+\;\tilde{\bar{G}}_2(k,u)\,\tilde{\varphi}_2(k)+\;\tilde{\bar{G}}_3(k,u)\,\tilde{\bar{f}}(k,u),\end{aligned} \tag{3.8}$$

where

$$\tilde{\bar{G}}_1(k,u) \;=\; \frac{u^{-\beta(2\alpha-1)-1}}{(u^{-2\alpha}+2au^{-\alpha}+dk^2)}\,, \tag{3.9}$$

$$\tilde{\bar{G}}_2(k,u) \;=\; \frac{2a\,u^{-\beta(\alpha-1)-1}}{(u^{-2\alpha}+2au^{-\alpha}+dk^2)}\,, \tag{3.10}$$

$$\tilde{\bar{G}}_3(k,u) \;=\; \frac{1}{(u^{-2\alpha}+2au^{-\alpha}+dk^2)}\,. \tag{3.11}$$

We know by the property of Fourier transform

$$e^{-c\,|\,x\,|}\;\overset{F}{\longleftrightarrow}\;\frac{2c}{c^2+k^2}, \tag{3.12}$$

we also have

$$\tilde{G}_1(x,u) \;=\; \frac{u^{-\beta(2\alpha-1)-1}}{2\sqrt{d(u^{-2\alpha}+2au^{-\alpha})}}\,e^{-\sqrt{((u^{-2\alpha}+2au^{-\alpha})/d)}\;|x|}, \tag{3.13}$$

$$\tilde{G}_2(x,p) \;=\; \frac{2a\,u^{-\beta(\alpha-1)-1}}{2\sqrt{d(u^{-2\alpha}+2au^{-\alpha})}}\,e^{-\sqrt{((u^{-2\alpha}+2au^{-\alpha})/d)}\;|x|}, \tag{3.14}$$

$$\tilde{G}_3(x,p) \;=\; \frac{1}{2\sqrt{d(u^{-2\alpha}+2au^{-\alpha})}}\,e^{-\sqrt{((u^{-2\alpha}+2au^{-\alpha})/d)}\;|x|}, \tag{3.15}$$

on solving, we get

$$\begin{aligned}U(x,t) \;=\; &\int_{-\infty}^{\infty}G_1(x-y,t)\varphi_1(y)dy+\int_{-\infty}^{\infty}G_2(x-y,t)\varphi_2(y)dy.\\ &.+\int_{-\infty}^{\infty}dy\int_0^t G_3(x-y,t-\tau)\,f(y,\tau)d\tau,\end{aligned} \tag{3.16}$$

where $G_1(x,t)$, $G_2(x,t)$and $G_3(x,t)$ are the corresponding fundamental solution obtained when $\varphi_1(x)=\varphi_2(x)=\delta(x),\; f(x)=0\;and\;\varphi(x)=0,\; f(x,t)=\delta(x)\,\delta(t)$ respectively, which is characterized by (3.6) or (3.7).

Now, we shall use following Sumudu transform pairs and one Fourier transform pair to express the Green function

$$F_1^{(\beta)}(t) \;:=\; \frac{t^\beta}{(\beta)!}\;\overset{S}{\longleftrightarrow}\;u^\beta, \tag{3.17}$$

$$F_2^{(\beta)}(ct) \;:=\; c\,w_\beta\,(ct)\;\overset{S}{\longleftrightarrow}\;e^{-(1/cu)^\beta}, \tag{3.18}$$

$$F_3\,(cx) \;:=\; \frac{1}{2\sqrt{\pi}}c^{-1/2}\,e^{-x^2/4c}\;\overset{F}{\longleftrightarrow}\;e^{-ck^2}, \tag{3.19}$$

where, $w_\beta (0<\beta<1)$ represents one-sided stable probability density, which can be explicitly expressed by Fox function

$$w_\beta(t) \;=\; \beta^{-1}\, t^{-2}\, H\ \begin{matrix}1,0\\1,1\end{matrix}\ \left(t^{-1}\left|{}^{-1,1}_{(-1/\beta,1/\beta)}\right.\right). \tag{3.20}$$

Then the Fourier-Sumudu transform of (3.7) can be rewritten as following integral form,

$$\tilde{\tilde{G}}_1(k,u) \;=\; u^{-\beta(2\alpha-1)-1}\int_0^\infty e^{-U(u^{-2\alpha}+2au^{-\alpha}+dk^2)}dU,$$
$$=\int_0^\infty [u^{-\beta(2\alpha-1)-1}].e^{-Uu^{-(2\alpha)}}e^{-Uu^{-2\alpha}}e^{-2aUu^{-\alpha}}e^{-Udk^2}dU,$$
$$=\int_0^\infty S\left\{\frac{t^{-\beta(2\alpha-1)-1}}{\Gamma(\beta-2\alpha\beta-2)}\right\}S\left\{F_2^{2\alpha}(U^{1/2}\alpha t)]\right\}S\left\{F_2^{\alpha}\left[(2aU)^{1/\alpha}\,t\right]\right\}\ .F\left\{F_3\,(dUx)\right\}dU,$$
$$=\int_0^\infty S\left[\left\{\frac{t^{-\beta(2\alpha-1)-1}}{\Gamma(\beta-2\alpha\beta-2)}\right\}*\left\{F_2^{2\alpha}(U^{1/2}\alpha t)]\right\}*\left\{F_2^{\alpha}\left[(2aU)^{1/\alpha}\,t\right]\right\}\right].F\left\{F_3\,(dUx)\right\}dU, \tag{3.21}$$

taking Fourier –Sumudu inverse transform, we obtain the following relation

$$=\int_0^y F\left\{F_3\,(dUx)\right\}\left(\int_0^t\left\{F_1^{(-\beta(2\alpha-1)-1)}\,(t-\tau)\right\}.\left\{F_2^{2\alpha}(U^{1/2}\alpha\tau).F_2^{\alpha}\left[(2aU)^{1/\alpha}\tau\right]\right\}d\tau\right)dU. \tag{3.22}$$

Again, working on the same lines, we may get

$$G_2(k,s)=2au^{-\beta(\alpha-1)-1}\int_0^y e^{-U(u^{-2\alpha}+2au^{-\alpha}+dk^2)}dU,$$
$$=2a\int_0^y [u^{-\beta(\alpha-1)-1}].e^{-Uu^{-2\alpha}}e^{-2aUu^{-\alpha}}e^{-Udk^2}dU,$$
$$=\ 2a\int_0^y S\left\{\frac{t^{-\beta(\alpha-1)-1}}{\Gamma(\beta-2\alpha\beta-2)}\right\}S\left\{F_2^{2\alpha}(U^{1/2}\alpha t)]\right\}S\left\{F_2^{\alpha}\left[(2aU)^{1/\alpha}t\right]\right\}\ .F\left\{F_3\,(dUx)\right\}dU,$$
$$=2a\int_0^\infty S\left[\left\{\frac{t^{-\beta(\alpha-1)-1}}{\Gamma(\beta-2\alpha\beta-2)}\right\}*\left\{F_2^{2\alpha}(U^{1/2}\alpha t)]\right\}*\left\{F_2^{\alpha}\left[(2aU)^{1/\alpha}\,t\right]\right\}\right].F\left\{F_3\,(dUx)\right\}dU, \tag{3.23}$$

taking Fourier –Sumudu inverse transform, we obtain the following relation

$$=2a\int_0^\infty F\left\{F_3\,(dUx)\right\}\left(\int_0^t\left\{F_1^{-\beta(\alpha-1)-1}\,(t-\tau)\right\}.\left\{F_2^{2\alpha}(U^{1/2}\alpha\tau).F_2^{\alpha}\left[(2aU)^{1/\alpha}\,\tau\right]\right\}d\tau\right)dU. \tag{3.24}$$

and for

$$\tilde{\tilde{G}}_3(k,u)=\int_0^\infty e^{-U(u^{-2\alpha}+2au^{-\alpha}+dk^2)}dU,$$
$$=\int_0^\infty e^{-Uu^{-2\alpha}}e^{-2au^{-\alpha}U}\,e^{-dk^2U}\ dU,$$
$$=\int_0^\infty S\left[\left\{F_2^{2\alpha}(U^{1/2}\alpha t)]\right\}*\left\{F_2^{\alpha}\left[(2aU)^{1/\alpha}\,t\right]\right\}\right]\ .F\left\{F_3\,(dUx)\right\}dU. \tag{3.25}$$

Going back to space-time domain, we obtain the relation

$$=\int_0^\infty F\left\{F_3\,(dUx)\right\}\left(\int_0^t\left\{F_2^{2\alpha}(U^{1/2}\alpha\tau).F_2^{\alpha}\left[(2aU)^{1/\alpha}\,(t-\tau)\right]\right\}d\tau\right)dU. \tag{3.26}$$

We can ensure that the green functions are non-negative by the non-negative properties of $F_2^{(\beta)}, F_3$. □

Theorem 3.2. *The Solution of the Generalized Signaling Problem* (3.4) *with initial condition*

$$I_t^{(1-\beta)(1-2\alpha)} U(x,0) = 0,$$

$$I_t^{(1-\beta)(1-\alpha)} U(x,0) = 0,$$

$$U(\mp\infty, t) = 0,$$

$$U(0,t) = g(t), \quad t > 0. \tag{3.27}$$

is given in the following proof.

Proof. By the application of Sumudu transform with f ≡ 0, and initial condition provided, we get

$$\frac{\partial^2 \tilde{U}(x,u)}{\partial x^2} = \frac{(u^{-2\alpha} + 2au^{-\alpha})}{d} \tilde{U}(x,u),$$

$$\tilde{U}(0,u) = \tilde{g}(u) \ , \tilde{U}(\infty,u) = 0, \tag{3.28}$$

with the solution

$$\tilde{U}(x,u) = g(u)\, e^{-\{\sqrt{(u^{-2\alpha}+2au^{-\alpha})/d}\}x} = S\{G(x,t) * g(t)\}, \tag{3.29}$$

where G(x,t) is the Green function or fundamental solution of the Signaling Problem obtained when g(x) = δ(x), which is characterized by

$$\tilde{G}(x,u) = e^{-\sqrt{((u^{-2\alpha}+2au^{-\alpha})/d)}\,x} \tag{3.30}$$

The inverse Sumudu transform of (3.29) provides the solution

$$U(x,t) = G(x,t) * g(t) = \int_0^t G(x,t-\tau)\, g(\tau)\, d\tau. \tag{3.31}$$

We can make relation by (3.13), (3.14) and (3.30)

$$\frac{\partial}{\partial s} G(x,u) = -2\alpha x G_1(x,u) - \alpha x G_2(x,u), \ x > 0. \tag{3.32}$$

Returning to the space-time domain we obtain the relation

$$t\, G(x,t) = 2\alpha x\, G_1(x,t) + \alpha x\, G_2(x,t)\ ,\ x,t > 0. \tag{3.33}$$

Then we can obtain a representation for G (x,t) and prove the negative properties. □

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Ravi Shanker Dubey

Department of Mathematics, Yagyavalkya Institute of Technology, Sitapura, Jaipur -302022,India.

E-mail address: `ravimath13@indiatimes.com`

Pranay Goswami

School of Liberal Studies, Bharat Ratna Dr. B. R. Ambedkar University, Delhi-110006, India.

E-mail address: `pranaygoswami83@gmail.com`

Fethi Bin Muhammad Belgacem

Department of Mathematics, Faculty of Basic Education, PAAET, Aardhia, Kuwait.

E-mail address: `fbmbelgacem@gmail.com`