Journal of Fractional Calculus and Applications,
Vol. 5(2) July 2014, pp. 9-16.
ISSN: 2090-5858.
http://fcag-egypt.com/Journals/JFCA/

EXISTENCE OF UNIQUE POSITIVE SOLUTION TO A TWO-POINT BOUNDARY-VALUE PROBLEM OF FRACTIONAL-ORDER SWITCHED SYSTEM WITH p-LAPLACIAN OPERATOR

XIANGSHAN KONG, DONGDONG WANG, HAITAO LI

Abstract. This work investigates the existence and uniqueness of a positive solution to a two-point boundary-value problem of fractional-order switched system with p-Laplacian operator, and presents a number of new results. First, the considered BVP is converted to an operator equation by using the property of Caputo derivative. Second, based on the operator equation and a fixed point theorem for a concave operator on a cone, a sufficient condition is presented for the existence and uniqueness of a positive solution. Finally, an illustrative example is given to support the obtained new results. The study of the illustrative example shows that the obtained results are effective.

1. Introduction

Fractional differential equations can properly describe many phenomena in various fields of science and engineering [1, 2, 3, 4] such as physics, technology, biology, chemical process, and so on. Due to this, the study of existence of solutions to various boundary value problems (BVPs) of fractional-order differential equations has attracted many scholars' interest [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. As an important branch of fractional-order differential equations, fractional-order p-Laplacian equations have been investigated in a series of recent works [20, 21, 22, 23, 24]. In [20], Chen and Liu considered the anti-periodic boundary value problem of fractional differential equation with p-Laplacian operator, and obtained the existence of one solution by using Schaefer's fixed point theorem under certain nonlinear growth conditions. Han et al. [21] investigated a class of fractional boundary-value problem with p-Laplacian operator and boundary parameter, and presented several existence results for a positive solution in terms of the boundary parameter.

While all the above results just considered a single-mode nonlinearity, fractional-order differential equations in practice often have switched nonlinearity, which is called switched systems. A switched system consists of a family of subsystems

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2000 *Mathematics Subject Classification.* 34A08, 34B18.

Key words and phrases. Positive solution, fractional-order switched system, Caputo fractional derivative, p-Laplacian operator.

Submitted Oct. 30, 2013 Revised Jan. 14, 2014.

described by differential or difference equations and a switching law that orchestrates switching between these subsystems. Switched systems arise as models for phenomena which cannot be described as exclusively continuous or exclusively discrete processes [25]. Due to their applications in traffic control, chemical processing, switching power converters, etc., switched systems have been studied by many scholars and lots of excellent results have been built up during the last three decades [26, 27, 28, 29, 30]. It is noted that the first issue of studying switched systems is whether or not the solution is unique. To the best of our knowledge, there is no paper available to answer this question for fractional-order switched system with p-Laplacian operator.

Motivated by the above, in the present paper, we study the following two-point boundary-value problem of fractional-order switched system with p-Laplacian operator:

$$\begin{cases} D^{\beta}_{0+}\Phi_p\Big(D^{\alpha}_{0+}x(t)\Big) = f_{\sigma(t)}(t, x(t)), t \in J = [0, 1], \\ x(0) = \gamma x(1), D^{\alpha}_{0+}x(0) = \eta D^{\alpha}_{0+}x(1), \end{cases} \tag{1}$$

where $\Phi_p(s) = |s|^{p-2}s,\ p > 1,\ \Phi_p^{-1} = \Phi_q,\ \frac{1}{p} + \frac{1}{q} = 1$, $0 < \alpha, \beta \leq 1$, $1 < \alpha + \beta \leq 2$, $0 < \gamma, \eta < 1$, $\sigma(t) : [0, 1] \to M = \{1, 2, \cdots, N\}$ is a finite switching signal which is a piecewise constant function depending on t, $R^+ = (0, +\infty)$, $f_i \in C[J \times R^+,\ R^+]$, $i \in M$ and D^{α}_{0+} is the Caputo derivative. Corresponding to the switching signal $\sigma(t)$, we have the following switching sequence:

$$\{(i_0,\ t_0),\ \cdots,\ (i_j,\ t_j),\ \cdots,\ (i_k,\ t_k)|i_j \in M,\ j = 0,\ 1,\ \cdots,\ k\}, \tag{2}$$

which means that the i_jth nonlinearity is activated when $t \in [t_j,\ t_{j+1})$ and the i_kth nonlinearity is activated when $t \in [t_k,\ 1]$. Here, $t_0 = 0$. Our purpose is to obtain sufficient conditions for the uniqueness of a positive solution to BVP (1). The main tool used in this work is a fixed point theorem for a concave operator on a cone. Throughout this paper, we consider BVP (1) in the real Banach space $E = C[0, 1]$ with the norm $\|x\| = \max_{t\in[0,1]} |x(t)|$. Let $P = \{x \in E : x(t) \geq 0,\ \forall\ t \in [0, 1]\}$. Then, P is a normal solid cone of E with $P^{\circ} = \{x \in E : x(t) > 0,\ \forall\ t \in [0, 1]\}$. A solution $x(t) \in E$ is said to be a positive solution to BVP (1), if $x \in P$ and $x(t) \not\equiv 0$. We study the existence of positive solutions to BVP (1) in P.

The main contributions of this paper are as follows. On one hand, we firstly investigate the existence of positive solutions to fractional-order switched systems with p-Laplacian operator, which enriches the theory of fractional-order differential equations. On the other hand, we propose a general method to deal with the switched nonlinearity, which can be used to study other kinds of BVPs of fractional-order switched systems.

The rest of this paper is structured as follows. Section 2 provides some necessary preliminaries which will be used to obtain the main results. Section 3 investigates the existence and uniqueness of a positive solution to BVP (1), which is followed by an illustrative example in Section 4.

2. Preliminaries

We give some necessary preliminaries on the Caputo derivative, which will be used in the sequel. For details, please refer to [2, 3, 4] and the references therein.

Definition 1 The Riemann-Liouville fractional integral of order $\alpha > 0$ of a function $y : (0, +\infty) \to \mathbb{R}$ is given by

$$I_{0+}^{\alpha} y(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} y(s) ds, \tag{3}$$

provided the right side is pointwise defined on $(0, +\infty)$.

Definition 2 The Caputo fractional derivative of order $\alpha > 0$ of a continuous function $y : (0, +\infty) \to \mathbb{R}$ is given by

$$D_{0+}^{\alpha} y(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{y^{(n)}(s)}{(t-s)^{\alpha-n+1}} ds, \tag{4}$$

where $n = [\alpha] + 1$, provided that the right side is pointwise defined on $(0, +\infty)$.

One can easily obtain the following property from the definition of Caputo derivative.

Proposition 1 Let $\alpha > 0$. Assume that $y, D_{0+}^{\alpha} y \in L(0, 1)$. Then the following equality holds:

$$I_{0+}^{\alpha} D_{0+}^{\alpha} y(t) = y(t) + C_0 + C_1 t + \cdots + C_{n-1} t^{n-1}, \tag{5}$$

for some $C_i \in \mathbb{R},\ i = 0, 1, \cdots, n-1$, where $n = [\alpha] + 1$.

3. Main results

In this section, we first convert BVP (1) into an equivalent operator equation, and then present some new results on the existence and uniqueness of positive solutions to BVP (1).

Lemma 1 Given $h \in C[0, 1]$, the unique solution of

$$\begin{cases} D_{0+}^{\beta} \Phi_p\Big(D_{0+}^{\alpha} x(t)\Big) = h(t), t \in [0, 1], \\ x(0) = \gamma x(1), D_{0+}^{\alpha} x(0) = \eta D_{0+}^{\alpha} x(1) \end{cases} \tag{6}$$

is

$$\begin{aligned} x(t) &= I_{0+}^{\alpha} \Phi_q\Big(I_{0+}^{\beta} h(t) + Ah(t)\Big) + Bh(t) \\ &= \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \Phi_q\Big(\frac{1}{\Gamma(\beta)} \int_0^s (s-\tau)^{\beta-1} h(\tau) d\tau + Ah(s)\Big) ds + Bh(t), \end{aligned}$$

where

$$\begin{aligned} Ah(t) &= \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)} I_{0+}^{\beta} h(t)|_{t=1} \\ &= \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)} \frac{1}{\Gamma(\beta)} \int_0^1 (1-s)^{\beta-1} h(s) ds,\ \forall\ t \in [0, 1], \end{aligned}$$

and

$$\begin{aligned} Bh(t) &= \frac{\gamma}{1-\gamma} I_{0+}^{\alpha} \Phi_q\Big(I_{0+}^{\beta} h(t) + Ah(t)\Big)|_{t=1} \\ &= \frac{\gamma}{1-\gamma} \frac{1}{\Gamma(\alpha)} \int_0^1 (1-s)^{\alpha-1} \Phi_q\Big(\frac{1}{\Gamma(\beta)} \int_0^s (s-\tau)^{\beta-1} h(\tau) d\tau + Ah(s)\Big) ds. \end{aligned}$$

Proof. Assume that $x(t)$ satisfies (6). Then, from Proposition 1 we have

$$\Phi_p\Big(D_{0+}^{\alpha} x(t)\Big) = I_{0+}^{\beta} h(t) + c_0,\ c_0 \in \mathbb{R}.$$

From the boundary value condition $D^{\alpha}_{0+}x(0) = \eta D^{\alpha}_{0+}x(1)$, one can see that

$$c_0 = \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)} I^{\beta}_{0+}h(t)|_{t=1} = Ah(t).$$

Thus, we have

$$x(t) = I^{\alpha}_{0+}\Phi_q\Big(I^{\beta}_{0+}h(t) + Ah(t)\Big) + c_1,\ c_1 \in \mathbb{R},$$

which together with the boundary value condition $x(0) = \gamma x(1)$ yields that

$$c_1 = \frac{\gamma}{1-\gamma} I^{\alpha}_{0+}\Phi_q\Big(I^{\beta}_{0+}h(t) + Ah(t)\Big)|_{t=1} = Bh(t).$$

The proof is completed. □

For any $x(t) \in C[0,1]$ and any switching sequence (2), define

$$\begin{aligned} Kx(t) &= I^{\alpha}_{0+}\Phi_q\Big(I^{\beta}_{0+}x(t) + Ax(t)\Big) + Bx(t) \\ &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\Phi_q\Big(\frac{1}{\Gamma(\beta)}\int_0^s (s-\tau)^{\beta-1}x(\tau)d\tau \\ &+ \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)}\frac{1}{\Gamma(\beta)}\int_0^1 (1-\tau)^{\beta-1}x(\tau)d\tau\Big)ds \\ &+ \frac{\gamma}{1-\gamma}\frac{1}{\Gamma(\alpha)}\int_0^1 (1-s)^{\alpha-1}\Phi_q\Big(\frac{1}{\Gamma(\beta)}\int_0^s (s-\tau)^{\beta-1}x(\tau)d\tau \\ &+ \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)}\frac{1}{\Gamma(\beta)}\int_0^1 (1-\tau)^{\beta-1}x(\tau)d\tau\Big)ds, \end{aligned} \tag{7}$$

and

$$F^{\sigma}(t,\ x) = \begin{cases} f_{i_0}(t,\ x), & t \in [0,\ t_1); \\ \vdots & \\ f_{i_j}(t,\ x), & t \in [t_j,\ t_{j+1}); \\ \vdots & \\ f_{i_k}(t,\ x), & t \in [t_k,\ 1]. \end{cases} \tag{8}$$

Lemma 2 $x(t) \in E$ is a solution to BVP (1), if and only if $x(t) = Tx(t)$, where $Tx(t) = K(F^{\sigma}(t, x(t)))$.

In the following, we study the existence of a unique positive solution to BVP (1). To this end, we need the following fixed point theorem [31].

Definition 3 Let P be a normal solid cone in a real Banach space E and P° be the interior of P. Suppose that $T : P^{\circ} \to P^{\circ}$ is an operator, and $0 \le \theta < 1$. Then T is called a θ-concave operator if

$$T(ku) \ge k^{\theta}Tu,\ \forall\ 0 < k < 1, u \in P^{\circ}.$$

Lemma 3 Assume that P is a normal solid cone in a real Banach space E, $0 \le \theta < 1$, and $T : P^{\circ} \to P^{\circ}$ is a θ-concave increasing operator. Then T has a unique fixed point in P°.

Now, we list some conditions on the nonlinearity of BVP (1).

(H1) For any $i \in M$, $f_i : J \times (0,\ +\infty) \to (0,\ +\infty)$.

(H2) For any $i \in M$, $f_i(t,x)$ is increasing in x for $x \in \mathbf{R}^+$.

(H3) For any $i \in M$, there exists a $\theta_i \in [0,\ 1)$, such that

$$f_i(t,kx) \ge k^{(p-1)\theta_i}f_i(t,x),\ \forall\ k \in (0,1),\ t \in J,\ x \in \mathbf{R}^+.$$

Remark 1 Conditions (H1)-(H3) imply the following conditions of $F^\sigma(t,x)$:
(H1′) $F^\sigma(t,x) > 0,\ \forall\ t \in J,\ x \in (0,\ +\infty)$.
(H2′) $F^\sigma(t,x)$ is increasing in x for $x \in \mathbf{R}^+$.
(H3′) $F^\sigma(t,kx) \geq k^{(p-1)\theta} F^\sigma(t,x),\ \forall\ k \in (0,1),\ t \in J,\ x \in \mathbf{R}^+$, where $\theta = \max\limits_{i\in M} \theta_i$.

Based on Lemma 3 and Remark 1, we have the following result.

Theorem 1 Suppose that (H1)-(H3) hold. Then for any finite switching signal $\sigma(t) : J \to M$, BVP (1) has a unique positive solution.

Proof. We first prove that $T : P^\circ \to P^\circ$.

For any $x \in P^\circ$, we have $x(t) > 0, t \in [0,1]$. Then (H1′) implies

$$
\begin{aligned}
Tx(t) &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\Phi_q\Big(\frac{1}{\Gamma(\beta)}\int_0^s (s-\tau)^{\beta-1}F^\sigma(\tau,x(\tau))d\tau \\
&+ \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)}\frac{1}{\Gamma(\beta)}\int_0^1 (1-\tau)^{\beta-1}F^\sigma(\tau,x(\tau))d\tau\Big)ds \\
&+ \frac{\gamma}{1-\gamma}\frac{1}{\Gamma(\alpha)}\int_0^1 (1-s)^{\alpha-1}\Phi_q\Big(\frac{1}{\Gamma(\beta)}\int_0^s (s-\tau)^{\beta-1}F^\sigma(\tau,x(\tau))d\tau \\
&+ \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)}\frac{1}{\Gamma(\beta)}\int_0^1 (1-\tau)^{\beta-1}F^\sigma(\tau,x(\tau))d\tau\Big)ds > 0,\ \forall\ t \in [0,1].
\end{aligned}
$$

Hence, $Tx \in P^\circ$.

Next, we prove that T is increasing in P°.

For any $x_1, x_2 \in P^\circ$ with $x_1 \leq x_2$, from the monotonicity of F^σ and x^{q-1}, we have $Tx_2(t) - Tx_1(t) \geq 0,\ \forall\ t \in [0,1]$, which implies that T is increasing in P°.

Finally, we prove that T is a θ-concave operator.

In fact, from (H3′), for any $0 < k < 1$, $x \in P^\circ$, it is easy to see that

$$
\begin{aligned}
T(kx)(t) &= \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\Phi_q\Big(\frac{1}{\Gamma(\beta)}\int_0^s (s-\tau)^{\beta-1}F^\sigma(\tau,kx(\tau))d\tau \\
&+ \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)}\frac{1}{\Gamma(\beta)}\int_0^1 (1-\tau)^{\beta-1}F^\sigma(\tau,kx(\tau))d\tau\Big)ds \\
&+ \frac{\gamma}{1-\gamma}\frac{1}{\Gamma(\alpha)}\int_0^1 (1-s)^{\alpha-1}\Phi_q\Big(\frac{1}{\Gamma(\beta)}\int_0^s (s-\tau)^{\beta-1}F^\sigma(\tau,kx(\tau))d\tau \\
&+ \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)}\frac{1}{\Gamma(\beta)}\int_0^1 (1-\tau)^{\beta-1}F^\sigma(\tau,kx(\tau))d\tau\Big)ds \\
&\geq k^\theta\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\Phi_q\Big(\frac{1}{\Gamma(\beta)}\int_0^s (s-\tau)^{\beta-1}F^\sigma(\tau,x(\tau))d\tau \\
&+ \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)}\frac{1}{\Gamma(\beta)}\int_0^1 (1-\tau)^{\beta-1}F^\sigma(\tau,x(\tau))d\tau\Big)ds \\
&+ k^\theta\frac{\gamma}{1-\gamma}\frac{1}{\Gamma(\alpha)}\int_0^1 (1-s)^{\alpha-1}\Phi_q\Big(\frac{1}{\Gamma(\beta)}\int_0^s (s-\tau)^{\beta-1}F^\sigma(\tau,x(\tau))d\tau \\
&+ \frac{\Phi_p(\eta)}{1-\Phi_p(\eta)}\frac{1}{\Gamma(\beta)}\int_0^1 (1-\tau)^{\beta-1}F^\sigma(\tau,x(\tau))d\tau\Big)ds \\
&= k^\theta T(x)(t),
\end{aligned}
$$

which shows that T is a θ-concave operator.

By Lemma 3, BVP (1) has a unique positive solution. □

4. An illustrative example

In this section, we give an illustrative example to support our new results.

Example 1 Consider the following BVP:

$$\begin{cases} D^{\beta}_{0+}\Phi_2\Big(D^{\alpha}_{0+}x(t)\Big) = f_{\sigma(t)}(t, x(t)), t \in J, \\ x(0) = \gamma x(1), D^{\alpha}_{0+}x(0) = \eta D^{\alpha}_{0+}x(1), \end{cases} \tag{9}$$

where $0 < \alpha, \beta \leq 1$, $1 < \alpha + \beta \leq 2$, $0 < \gamma, \eta < 1$ are arbitrary, $\sigma(t) : J \to M = \{1,\ 2,\ 3\}$ is a finite switching signal, and

$$f_1(t,\ x) = (1+t)\sqrt{x},\ f_2(t,\ x) = (2+t^2)x^{\frac{2}{3}},\ f_3(t,\ x) = (3+t^3)x^{\frac{3}{4}}.$$

It is easy to see that

$$f_i(t,\ x) > 0,\ \forall\ t \in J,\ x \in (0,\ +\infty),\ i = 1,\ 2,\ 3.$$

Thus, (H1) holds.

A simple calculation shows that

$$\frac{\partial f_1(t,\ x)}{\partial x} = \frac{1+t}{2\sqrt{x}} > 0,\ \forall\ t \in J,\ x \in (0,\ +\infty),$$

$$\frac{\partial f_2(t,\ x)}{\partial x} = \frac{2(2+t^2)}{3\sqrt[3]{x}} > 0,\ \forall\ t \in J,\ x \in (0,\ +\infty),$$

and

$$\frac{\partial f_3(t,\ x)}{\partial x} = \frac{3(3+t^3)}{4\sqrt[4]{x}} > 0,\ \forall\ t \in J,\ x \in (0,\ +\infty).$$

Hence, (H2) is satisfied.

Now we check (H3). In fact,

$$f_1(t,\ kx) = \sqrt{k}(1+t)\sqrt{x} \geq k^{\frac{1}{2}} f_1(t,\ x),\ \forall\ k \in (0,1),\ t \in J,\ x \in \mathbf{R}^+,$$

$$f_2(t,\ kx) = k^{\frac{2}{3}}(2+t^2)x^{\frac{2}{3}} \geq k^{\frac{2}{3}} f_2(t,\ x),\ \forall\ k \in (0,1),\ t \in J,\ x \in \mathbf{R}^+,$$

and

$$f_3(t,\ kx) = k^{\frac{3}{4}}(3+t^3)x^{\frac{3}{4}} \geq k^{\frac{3}{4}} f_3(t,\ x),\ \forall\ k \in (0,1),\ t \in J,\ x \in \mathbf{R}^+.$$

Therefore, (H3) holds.

Hence, Theorem 1 shows that for any finite switching signal $\sigma(t) : J \to M$, BVP (9) has a unique positive solution.

Acknowledgements

The authors would like to thank the anonymous reviewer for his/her constructive comments and suggestions which improved the quality of the paper.

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Xiangshan Kong
Basic Science Department, Qingdao Binhai University, Qingdao, P.R. China
E-mail address: kong_xiangshan@126.com

Dongdong Wang
Basic Science Department, Qingdao Binhai University, Qingdao, P.R. China
E-mail address: hotmm2006@126.com

Haitao Li
School of Control Science and Engineering, Shandong University, Jinan, P.R. China
E-mail address: haitaoli09@gmail.com