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SOME BASIC PROPERTIES OF EULER TYPE INTEGRAL OPERATORS INVOLVING WITH GENERALIZED $k-$WRIGHT FUNCTION

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Abstract. This paper deals with the k-extension of the Wright's generalized hyper-geometric function. In this paper, authors study fractional order Euler type integral operators involving the generalized k-Wright function defined by Gehlot and Prajapati [5]. Some special cases of the main results are also investigated.

1. Introduction

For $z \in \mathbb{C}$ and $k \in \mathbb{R}^+$, the k-extension of the Wright generalized hypergeometric function ${}_p\Psi_q(z)$ was introduced by Gehlot and Prajapati [5] in 2013 and which known as generalized k-Wright function and defined as $\alpha_i, \beta_j \in \mathbb{C}$, $A_i, B_j \in \mathbb{R}$ $(A_i, B_j \neq 0;\ i = 1, ..., p;\ j = 1, ..., q)$ and $(\alpha_i + A_i n), (\beta_j + B_j n) \in \mathbb{C} \setminus k\mathbb{Z}^-$.

$$ {}_p\Psi_q^k(z) = {}_p\Psi_q^k \left[\begin{array}{c} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; z \right] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q} \Gamma_k(\beta_j + B_j n)} \frac{z^n}{n!} \tag{1} $$

(i). If $\Delta > -1$, then series (1) is absolutely convergent for all $z \in \mathbb{C}$ and generalized k-Wright function ${}_p\Psi_q^k(z)$ is an entire function of z.

(ii). If $\Delta = -1$, then series (1) is absolutely convergent for all $z < \Omega$ and if $z = \Omega$ then $Re(\Theta) > -\frac{1}{2}$, where Δ, Ω and Θ are given by

$$ \Delta = \sum_{j=1}^{q} \left(\frac{B_j}{k} \right) - \sum_{i=1}^{p} \left(\frac{A_i}{k} \right), $$

$$ \Omega = \prod_{i=1}^{p} \left| \frac{A_i}{k} \right|^{-\frac{A_i}{k}} \prod_{j=1}^{q} \left| \frac{B_j}{k} \right|^{\frac{B_j}{k}}, $$

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$$\Theta = \sum_{j=1}^{q} \left(\frac{\beta_j}{k}\right) - \sum_{i=1}^{p} \left(\frac{\alpha_i}{k}\right) + \frac{p-q}{2},$$

particular $k = 1$, the generalized k-Wright function reduced to Wright generalized hypergeometric function ${}_p\Psi_q(z)$ which was introduced by Wright [17]

In recent years, k-extensions of well-known special functions of mathematical physics have been explored by several authors. (see [6],[2],[5],[15]). Diaz and Pariguan [3] have introduced k-extensions of the gamma and beta functions where $k > 0$. Mubeen at el.[13] introduced the extended k-gamma and extended k-beta functions with their primary properties.We will now use the following definitions to derive the main results in our work.

2. Definitions

Definition 2.1 The integral representation of gamma function and beta function defined [3] as

$$\Gamma(x) = \int_0^\infty \xi^{x-1} e^{-\xi}\, d\xi; \quad Re(x) > 0 \tag{2}$$

$$B(x,y) = \int_0^1 \xi^{x-1}(1-\xi)^{y-1} d\xi; \quad Re(x) > 0,\ Re(y) > 0 \tag{3}$$

Definition 2.2 For $Re(x) > 0$ and $Re(y) > 0$, the following integral formula introduced by MacRobert [10] and defined as

$$B(x,y) = A^x B^y \int_0^1 \frac{\xi^{x-1}(1-\xi)^{y-1}}{\{A\xi + B(1-\xi)\}^{x+y}} d\xi; \quad A \neq 0, B \neq 0 \tag{4}$$

Definition 2.3 For $Re(x) > 0$ and $Re(y) > 0$, the following integral formula introduced by Lavoie-Trottier [9] and defined as

$$B\,(x,y) = \left(\frac{3}{2}\right)^{2x} \int_0^1 \xi^{x-1}\,(1-\xi)^{2y-1} \left(1 - \frac{\xi}{3}\right)^{2x-1} \left(1 - \frac{\xi}{4}\right)^{y-1} d\xi\,; \tag{5}$$

Definition 2.4 For $k > 0$ and $x \in \mathbb{C}$, the k-gamma function is defined [3] as

$$\Gamma_k(x) = \lim_{n\to\infty} \frac{n! k^n (nk)^{\frac{x}{k}-1}}{(x)_{n,k}}; \tag{6}$$

where $(x)_{n,k}$ is Pochhammer k-symbol and given by $(x)_{n,k} = \frac{\Gamma_k(x+nk)}{\Gamma_k(x)}$.

Definition 2.5 The integral representation of k-gamma function and k-beta function defined [13] as for $k > 0$

$$\Gamma_k(x) = \int_0^\infty \xi^{x-1} e^{\frac{\xi^k}{k}} d\xi; \quad Re(x) > 0 \tag{7}$$

$$B_k(x,y) = \frac{1}{k} \int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{y}{k}-1} d\xi; \quad Re(x) > 0,\ Re(y) > 0 \tag{8}$$

Definition 2.6 The k-analogue of the extended Eulers beta function [13] is defined as for $k > 0$ and $Re(A) > 0$

$$B_k\left(x, y; A\right) = \frac{1}{k}\int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{y}{k}-1}e^{-\frac{A^k}{k\xi(1-\xi)}}d\xi; \quad Re(x) > 0, Re(y) > 0 \tag{9}$$

and if $A = 0$ then $B_k(x, y; 0)$ tends to $B_k(x, y)$ and the relation between k-gamma, k-beta and Eulers beta function is given by the following formula

$$B_k(x, y) = \frac{1}{k}B\left(\frac{x}{k}, \frac{y}{k}\right) = \frac{\Gamma_k(x)\Gamma_k(y)}{\Gamma_k(x+y)}; \tag{10}$$

3. Main Results

Theorem 3.1 If the condition (1) is satisfied and $B_k\left(x+\alpha n,\, y; A\right)$ is the k-analogue of the extended Euler beta function then the following integral hold true.

$$\int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{y}{k}-1}exp\left(\frac{-A^k}{k\xi(1-\xi)}\right)\, {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array}; z\xi^{\frac{\alpha}{k}}\right]d\xi$$

$$= k \times \left\{{}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array}; z\right] \otimes B_k\left(x+\alpha n,\, y; A\right)\right\} \tag{11}$$

where $\otimes$ stands for convolution product over summation $n = 0$ to ∞ .

Proof. In order to derive (11), we denote L.H.S. of (11) by symbol I_1 and then expanding ${}_p\Psi_q^k(z\xi^{\frac{\alpha}{k}})$ by using equation (1),

$$I_1 \equiv \int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{y}{k}-1}exp\left(\frac{-A^k}{k\xi(1-\xi)}\right)\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j + B_j n)}\frac{z^n\xi^{\frac{\alpha n}{k}}}{n!}d\xi; \tag{12}$$

by changing the order of summation and integration, we have

$$I_1 \equiv \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j + B_j n)}\frac{z^n}{n!}\int_0^1 \xi^{\frac{x+\alpha n}{k}-1}(1-\xi)^{\frac{y}{k}-1}exp\left(\frac{-A^k}{k\xi(1-\xi)}\right)d\xi; \tag{13}$$

now using the following k-analogue of the extended Eulers beta function, given by Mubeen et al. [13], we arrive

$$B_k\left(\lambda, \delta; A\right) = \frac{1}{k}\int_0^1 \theta^{\frac{\lambda}{k}-1}(1-\theta)^{\frac{\delta}{k}-1}e^{-\frac{A^k}{k\theta(1-\theta)}}d\theta \tag{14}$$

$$I_1 \equiv k \times \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j + B_j n)}\frac{z^n}{n!}B_k\left(x+\alpha n,\, y; A\right); \tag{15}$$

$$I_1 \equiv k \times \left\{{}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array}; z\right] \otimes B_k\left(x+\alpha n,\, y; A\right)\right\}. \tag{16}$$

Theorem 3.2 If the condition (1) is satisfied and $B_k\left(x+kn,\, y+kn\right)$ is the k-analogue of Euler beta function then the following integral hold true.

$$\int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{y}{k}-1}\left\{A\xi + B(1-\xi)\right\}^{-\frac{x+y}{k}}\, {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array}; \frac{z\,\xi(1-\xi)}{\left\{A\xi + B(1-\xi)\right\}^2}\right]d\xi$$

$$= k \times \frac{1}{A^{\frac{x}{k}} B^{\frac{y}{k}}} \left\{ {}_p\Psi_q^k \left[\begin{array}{l} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; \frac{z}{AB} \right] \otimes B_k\left(x + kn,\ y + kn\right) \right\} \tag{17}$$

where $\otimes$ stands for convolution product over summation $n = 0$ to ∞ .

Proof. In order to derive (17), we denote L.H.S. of (17) by symbol I_2 and then expanding ${}_p\Psi_q^k \left(\frac{z\,\xi(1-\xi)}{\{A\xi + B(1-\xi)\}^2} \right)$ by using equation (1),

$$I_2 \equiv \int_0^1 \frac{\xi^{\frac{x}{k}-1}(1-\xi)^{\frac{y}{k}-1}}{\{A\xi + B(1-\xi)\}^{\frac{x+y}{k}}} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q} \Gamma_k(\beta_j + B_j n)} \frac{z^n \xi^n (1-\xi)^n}{\{A\xi + B(1-\xi)\}^{2n}\, n!} d\xi; \tag{18}$$

by interchanging the order of summation and integration, we have

$$I_2 \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q} \Gamma_k(\beta_j + B_j n)} \frac{z^n}{n!} \int_0^1 \frac{\xi^{\frac{x}{k}+n-1}(1-\xi)^{\frac{y}{k}+n-1}}{\{A\xi + B(1-\xi)\}^{\frac{x+y}{k}+2n}} d\xi; \tag{19}$$

now using the following result given by MacRobert [10] and after some simplification,

$$\int_0^1 \frac{\xi^{\lambda-1}(1-\xi)^{\delta-1}}{\{A\xi + B(1-\xi)\}^{\lambda+\delta}} d\xi = \frac{B(\lambda, \delta)}{A^{\lambda} B^{\delta}} \tag{20}$$

$$I_2 \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q} \Gamma_k(\beta_j + B_j n)} \frac{z^n}{n!} \frac{B\left(\frac{x}{k} + n, \frac{y}{k} + n\right)}{A^{\frac{x}{k}+n} B^{\frac{y}{k}+n}}; \tag{21}$$

$$I_2 \equiv k \times \frac{1}{A^{\frac{x}{k}} B^{\frac{y}{k}}} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q} \Gamma_k(\beta_j + B_j n)} \frac{\left(\frac{z}{AB}\right)^n}{n!} B_k\left(x + kn,\ y + kn\right); \tag{22}$$

$$I_2 \equiv k \times \frac{1}{A^{\frac{x}{k}} B^{\frac{y}{k}}} \left\{ {}_p\Psi_q^k \left[\begin{array}{l} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; \frac{z}{AB} \right] \otimes B_k\left(x + kn,\ y + kn\right) \right\}. \tag{23}$$

Theorem 3.3 If the condition (1) is satisfied and $B_k\left(x,\ y + n\right)$ is the k-analogue of Euler beta function then the following integral hold true.

$$\int_0^1 \xi^{\frac{x}{k}-1} (1-\xi)^{\frac{2y}{k}-1} \left(1 - \frac{\xi}{3}\right)^{\frac{2x}{k}-1} \left(1 - \frac{\xi}{4}\right)^{\frac{y}{k}-1} {}_p\Psi_q^k \left[\begin{array}{l} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; z\,(1-\xi)^2 \left(1 - \frac{\xi}{4}\right) \right] d\xi$$

$$= k \times \left(\frac{4}{9}\right)^{\frac{x}{k}} \left\{ {}_p\Psi_q^k \left[\begin{array}{l} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; z \right] \otimes B_k\left(x,\ y + kn\right) \right\} \tag{24}$$

where $\otimes$ stands for convolution product over summation $n = 0$ to ∞ .

Proof. In order to derive (24), we denote L.H.S. of (24) by symbol I_3 and then expanding ${}_p\Psi_q^k \left[z(1-\xi)^{\frac{2}{k}} \left(1 - \frac{\xi}{4}\right)^{\frac{1}{k}} \right]$ by using equation (1),

$$I_3 \equiv \int_0^1 \xi^{\frac{x}{k}-1} (1-\xi)^{\frac{2y}{k}-1} \left(1 - \frac{\xi}{3}\right)^{\frac{2x}{k}-1} \left(1 - \frac{\xi}{4}\right)^{\frac{y}{k}-1}$$

$$\times \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q} \Gamma_k(\beta_j + B_j n)} \frac{z^n}{n!} (1-\xi)^{2n} \left(1 - \frac{\xi}{4}\right)^n d\xi; \tag{25}$$

by interchanging the order of summation and integration, we have

$$I_3 \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k(\alpha_i + A_i n)}{\prod_{j=1}^{q} \Gamma_k(\beta_j + B_j n)} \frac{z^n}{n!} \int_0^1 \xi^{\frac{x}{k}-1} (1-\xi)^{\frac{2y}{k}+2n-1} \left(1 - \frac{\xi}{3}\right)^{\frac{2x}{k}-1} \left(1 - \frac{\xi}{4}\right)^{\frac{y}{k}+n-1} d\xi; \tag{26}$$

now using the following result given by Lavoie-Trottier [9] and after some simplification,

$$\int_0^1 \xi^{\lambda-1}(1-\xi)^{2\delta-1}\left(1-\frac{\xi}{3}\right)^{2\lambda-1}\left(1-\frac{\xi}{4}\right)^{\delta-1} d\xi = \left(\frac{4}{9}\right)^{\lambda} B(\lambda,\delta) \tag{27}$$

$$I_3 \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i+A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j+B_j n)} \frac{z^n}{n!} \left(\frac{4}{9}\right)^{\frac{x}{k}} B\left(\frac{x}{k}, \frac{y+kn}{k}\right); \tag{28}$$

$$I_3 \equiv k \times \left(\frac{4}{9}\right)^{\frac{x}{k}} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i+A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j+B_j n)} \frac{z^n}{n!} B_k(x,\ y+kn); \tag{29}$$

$$I_3 \equiv k \times \left(\frac{4}{9}\right)^{\frac{x}{k}} \left\{ {}_p\Psi_q^k \left[\begin{array}{l} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; z \right] \otimes B_k(x,\ y+kn) \right\}. \tag{30}$$

Theorem 3.4 If the condition (1) is satisfied and $B_k(x+n,\ y)$ is the k-analogue of Euler beta function then the following integral hold true.

$$\int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{2y}{k}-1}\left(1-\frac{\xi}{3}\right)^{\frac{2x}{k}-1}\left(1-\frac{\xi}{4}\right)^{\frac{y}{k}-1} {}_p\Psi_q^k \left[\begin{array}{l} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; z\,\xi\left(1-\frac{\xi}{3}\right)^2 \right] d\xi$$

$$= k \times \left(\frac{4}{9}\right)^{\frac{x}{k}} \left\{ {}_p\Psi_q^k \left[\begin{array}{l} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; \frac{4z}{9} \right] \otimes B_k(x+kn,\ y) \right\} \tag{31}$$

where $\otimes$ stands for convolution product over summation $n = 0$ to ∞ .

Proof. In order to derive (31), we denote L.H.S. of (31) by symbol I_4 and then expanding ${}_p\Psi_q^k\left[z\,\xi^{\frac{1}{k}}\left(1-\frac{\xi}{3}\right)^{\frac{2}{k}}\right]$ by using equation (1),

$$I_4 \equiv \int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{2y}{k}-1}\left(1-\frac{\xi}{3}\right)^{\frac{2x}{k}-1}\left(1-\frac{\xi}{4}\right)^{\frac{y}{k}-1}$$

$$\times \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i+A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j+B_j n)} \frac{z^n\,\xi^n}{n!} \left(1-\frac{\xi}{3}\right)^{2n} d\xi; \tag{32}$$

by interchanging the order of summation and integration, we have

$$I_4 \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i+A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j+B_j n)} \frac{z^n}{n!} \int_0^1 \xi^{\frac{x}{k}+n-1}(1-\xi)^{\frac{2y}{k}-1}\left(1-\frac{\xi}{3}\right)^{\frac{2x}{k}+2n-1}\left(1-\frac{\xi}{4}\right)^{\frac{y}{k}-1} d\xi; \tag{33}$$

now using the following result given by Lavoie-Trottier [9] and further simplification,

$$\int_0^1 \xi^{\lambda-1}(1-\xi)^{2\delta-1}\left(1-\frac{\xi}{3}\right)^{2\lambda-1}\left(1-\frac{\xi}{4}\right)^{\delta-1} d\xi = \left(\frac{4}{9}\right)^{\lambda} B(\lambda,\delta) \tag{34}$$

$$I_4 \equiv \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i+A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j+B_j n)} \frac{z^n}{n!} \left(\frac{4}{9}\right)^{\frac{x}{k}+n} B\left(\frac{x+kn}{k}, \frac{y}{k}\right); \tag{35}$$

$$I_4 \equiv k \times \left(\frac{4}{9}\right)^{\frac{x}{k}} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma_k(\alpha_i+A_i n)}{\prod_{j=1}^{q}\Gamma_k(\beta_j+B_j n)} \frac{\left(\frac{4z}{9}\right)^n}{n!} B_k(x+kn,\ y); \tag{36}$$

$$I_4 \equiv k \times \left(\frac{4}{9}\right)^{\frac{x}{k}} \left\{ {}_p\Psi_q^k \left[\begin{array}{l} (\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q} \end{array} ; \frac{4z}{9} \right] \otimes B_k(x+kn,\ y) \right\}. \tag{37}$$

4. Special Cases

In this section, we establish the following useful integral operators of fractional calculus involving the generalized k-Wright function as special cases of our main results.

1. Setting $A = 0$ in Theorem 3.1, we attain:

$$\int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{y}{k}-1}\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};z\,\xi^{\frac{\alpha}{k}}\right]d\xi$$
$$= k \times\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};z\right]\otimes B_k\left(x+\alpha n,\ y\right). \tag{38}$$

2. Setting $A = B = 1$ in Theorem 3.2, we obtain:

$$\int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)^{\frac{y}{k}-1}\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};z\,\xi(1-\xi)\right]d\xi$$
$$= k \times\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};z\right]\otimes B_k\left(x+kn,\ y+kn\right). \tag{39}$$

3. Setting $x = k$ in Theorem 3.2, we get:

$$\int_0^1 (1-\xi)^{\frac{y}{k}-1}\left\{A\xi+B(1-\xi)\right\}^{-\frac{y}{k}-1}\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};\frac{z\,\xi(1-\xi)}{\left\{A\xi+B(1-\xi)\right\}^2}\right]d\xi$$
$$= k \times \frac{1}{AB^{\frac{y}{k}}}\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};\frac{z}{AB}\right]\otimes B_k\left(k+kn,\ y+kn\right). \tag{40}$$

4. Setting $y = k$ in Theorem 3.2, we find:

$$\int_0^1 \xi^{\frac{x}{k}-1}\left\{A\xi+B(1-\xi)\right\}^{-\frac{x}{k}-1}\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};\frac{z\,\xi(1-\xi)}{\left\{A\xi+B(1-\xi)\right\}^2}\right]d\xi$$
$$= k \times \frac{1}{A^{\frac{x}{k}}B}\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};\frac{z}{AB}\right]\otimes B_k\left(x+kn,\ k+kn\right). \tag{41}$$

5. Setting $x = k$ in Theorem 3.3, we get:

$$\int_0^1 (1-\xi)^{\frac{2y}{k}-1}\left(1-\frac{\xi}{3}\right)\left(1-\frac{\xi}{4}\right)^{\frac{y}{k}-1}\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};z\,(1-\xi)^2\left(1-\frac{\xi}{4}\right)\right]d\xi$$
$$= k \times \left(\frac{4}{9}\right)\ {}_p\Psi_q^k\left[\begin{array}{l}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{array};z\right]\otimes B_k\left(k,\ y+kn\right). \tag{42}$$

6. Setting $y = k$ in Theorem 3.3, we find:

$$\int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)\left(1-\frac{\xi}{3}\right)^{\frac{2x}{k}-1} {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};z(1-\xi)^2\left(1-\frac{\xi}{4}\right)\right]d\xi$$

$$= k \times \left(\frac{4}{9}\right)^{\frac{x}{k}} {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};z\right]\otimes B_k(x,\ k+kn). \tag{43}$$

7. Setting $x = y = k$ in Theorem 3.3, we achieve:

$$\int_0^1 (1-\xi)\left(1-\frac{\xi}{3}\right) {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};z(1-\xi)^2\left(1-\frac{\xi}{4}\right)\right]d\xi$$

$$= k \times \left(\frac{4}{9}\right) {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};z\right]\otimes B_k(k,\ k+kn). \tag{44}$$

8. Setting $x = k$ in Theorem 3.4, we get:

$$\int_0^1 (1-\xi)^{\frac{2y}{k}-1}\left(1-\frac{\xi}{3}\right)\left(1-\frac{\xi}{4}\right)^{\frac{y}{k}-1} {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};z\ \xi\left(1-\frac{\xi}{3}\right)^2\right]d\xi$$

$$= k \times \left(\frac{4}{9}\right) {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};\frac{4z}{9}\right]\otimes B_k(k+kn,\ y). \tag{45}$$

9. Setting $y = k$ in Theorem 3.4, we find:

$$\int_0^1 \xi^{\frac{x}{k}-1}(1-\xi)\left(1-\frac{\xi}{3}\right)^{\frac{2x}{k}-1} {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};z\ \xi\left(1-\frac{\xi}{3}\right)^2\right]d\xi$$

$$= k \times \left(\frac{4}{9}\right)^{\frac{x}{k}} {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};\frac{4z}{9}\right]\otimes B_k(x+kn,\ k). \tag{46}$$

10. Setting $x = y = k$ in Theorem 3.4, we achieve:

$$\int_0^1 (1-\xi)\left(1-\frac{\xi}{3}\right) {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};z\ \xi\left(1-\frac{\xi}{3}\right)^2\right]d\xi$$

$$= k \times \left(\frac{4}{9}\right) {}_p\Psi_q^k\left[\begin{matrix}(\alpha_i, A_i)_{1,p}\\(\beta_j, B_j)_{1,q}\end{matrix};\frac{4z}{9}\right]\otimes B_k(k+kn,\ k). \tag{47}$$

Conclusion

In this article we have established some Euler type integral formulas involving the generalized k-Wright function and the result obtained is the product of the k-Wright function and the extended k-beta function. We believe that the results of this paper will be significant in the theory of fractional k-calculus and especially taking $k = 1$ it becomes the results of the ordinary calculus.

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