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A WAVELET-BASED COLLOCATION METHOD FOR FRACTIONAL CAHN-ALLEN EQUATIONS

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Abstract. In the numerical analysis, wavelets play an important role in dealing with approximate solutions of differential equations. Haar wavelet basis permits to enlarge a class of functions in the collocation frame work. In this article, we propose the Haar wavelet-based numerical technique for solving non-linear fractional Cahn-Allen equations. Through the proposed technique, the solution is obtained on the coarse grid points and then refined towards higher accuracy by increasing the level of the Haar wavelets. The advantages of the proposed technique are the sparse structure of the Haar wavelets matrices, the small computation costs, the small number of significant wavelets coefficients and the simple applicability for a variety of boundary conditions. The solution process, the efficiency and the applicability of the proposed technique are demonstrated by an example.

1. Introduction

Fractional calculus is a generalization of ordinary calculus to an arbitrary order. In recent years, several fractional models have drawn much attention in diverse disciplines of science and technology, such as viscoelasticity, diffusion, control theory, electromagnetism, electrochemistry, biosciences, bioengineering, fluid mechanics, non-linear dynamical systems and so on.

However, analytical solutions do not exist for most of the Fractional Differential Equations(FDEs). Owing to this fact, many numerical techniques have been introduced to find approximate solutions of FDEs. These numerical techniques include Adomian decomposition method[14], Laplace Adomian decomposition method[16], Natural reduced differential transform method[7, 9], General fractional residual power series method[8], Variational iteration transform method[28], Homotopy perturbation method[24], Homotopy analysis method[4], Homotopy perturbation transform method[23] and wavelet based numerical method for inverse Laplace transform[1].

Wavelets' bases have received much attention in dealing with approximate solutions of differential equations with integer and non-integer orders. Wavelets permit

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an accurate representation for a variety of functions and so establish a connection with the fast numerical algorithms. The prominent attributes of wavelets are to make out singularities, irregular structure and transient phenomena exhibited by the analyzed equations. The wavelet-based techniques for solving differential equations usually depend either on the collocation methods or on the Galerkin methods. Haar wavelets are the simplest orthonormal compact supported piecewise constant functions. As the derivatives of Haar wavelets do not exist at the breaking points, it is not possible to solve partial differential equations by Haar wavelets directly. There are mainly two possibilities to get out of this hurdle. By the first way, Haar wavelets can be regularized with interpolating splines. This approach was applied by Cattani[2]. The another way is to make use of the integral method that was introduced by Chen and Hsiao[3]. Recently, Lepik [10, 11, 12, 13] has setup a technique based on Haar wavelets for solving differential equations.

The non-linear fractional Cahn-Allen equations arise in many fields of physical phenomena such as quantum mechanics, plasma physics, gas dynamics and mathematical biology. Many authors have investigated the Cahn-Allen equation using various methodologies in recent years. In[4], A. Esen et al. solved the time fractional Cahn-Allen equations by Homotopy analysis method. A. Prakash and H. kaur[19] investigated a numerical approach for time fractional Cahn-Allen equations. In[20], M. S. Rawashdeh employed the fractional reduced differential transform method to solve time fractional Cahn-Allen equations. An iterative reproducing kernel method is introduced for the time-fractional CahnAllen equation by M.G. Sakar et al.[21].

In this article, we demonstrate the Haar wavelet-based numerical technique for solving fractional Cahn-Allen equations with initial and boundary conditions. This wavelet based numerical technique is very compatible to solve boundary value problems.

This article is presented in the following way. Some basic definitions and mathematical preliminary facts of fractional calculus are discussed in section 2. Section 3 is devoted to the basic formulation of Haar wavelets, function approximation, fractional integration of Haar wavelets and fractional integration matrix of Haar wavelets. In section 4, the proposed wavelet-based numerical technique for solving fractional Cahn-Allen equation is discussed and we also demonstrate the applicability of the proposed numerical technique by considering a numerical example. Finally we conclude our work in section 5.

2. Preliminaries

In this section, some necessary definitions and mathematical preliminary facts of fractional calculus are discussed.

Definition 1 [15, 17, 18, 25] Let $\theta = (\theta_1, \theta_2) \in (0, \infty) \times (0, \infty)$ and $h(x,t) \in L^1([0,a] \times [0,b])$, $a, b > 0$. The mixed Riemann-liouville fractional integral of order θ of $h(x,t)$ is defined as

$$J^{\theta} h(x,t) = \frac{1}{\Gamma(\theta_1)\Gamma(\theta_2)} \int_0^x \int_0^t (x-\upsilon)^{\theta_1 - 1}(t-\tau)^{\theta_2 - 1} h(\upsilon, \tau) d\tau d\upsilon,$$

whenever the integral exists.

The Riemann-liouville fractional integral of order $\theta_1 > 0$, along x, keeping t constant, is defined as

$$J_x^{\theta_1} h(x,t) = \frac{1}{\Gamma(\theta_1)} \int_0^x (x-\upsilon)^{\theta_1 - 1} h(\upsilon, t) d\upsilon, \text{ whenever the integral exists.}$$

The Riemann-liouville fractional integral of order $\theta_2 > 0$, along t, keeping x constant, is defined as

$$J_t^{\theta_2} h(x,t) = \frac{1}{\Gamma(\theta_2)} \int_0^t (t-\tau)^{\theta_2 - 1} h(x, \tau) d\tau, \text{ whenever the integral exists.}$$

Let $h(x,t)$, $g(x,t) \in L^1([0,a] \times [0,b])$, $a, b > 0$, λ, $\gamma \in \mathbb{R}$, $\nu, \mu > -1$.
Then the following properties are attained.

(i) $J^\theta(\lambda h(x,t) + \gamma g(x,t)) = \lambda J^\theta h(x,t) + \gamma J^\theta g(x,t)$.

(ii) $J^\theta x^\nu t^\mu = \dfrac{\Gamma(\nu+1)\Gamma(\mu+1)}{\Gamma(\nu+\theta_1+1)\Gamma(\mu+\theta_2+1)} x^{\theta_1+\nu} t^{\theta_2+\mu}$.

Definition 2 Let $\theta = (\theta_1, \theta_2) \in (0,1] \times (0,1]$ and $h(x,t) \in L^1([0,a] \times [0,b])$, $a, b > 0$. The Caputo fractional derivative of order θ of $h(x,t)$ is defined as

$$D^\theta h(x,t) = \frac{1}{\Gamma(1-\theta_1)\Gamma(1-\theta_2)} \int_0^x \int_0^t (x-\upsilon)^{-\theta_1} (t-\tau)^{-\theta_2} \frac{\partial^2 h(\upsilon,\tau)}{\partial\upsilon \partial\tau} d\tau d\upsilon.$$

The Caputo derivative of order $\alpha > 0$, along x, keeping t constant, is defined as

$$D_x^\alpha h(x,t) = \frac{1}{\Gamma(m-\alpha)} \int_0^x (x-\upsilon)^{m-\alpha-1} \frac{\partial^m h(\upsilon,t)}{\partial \upsilon^m} d\upsilon, \ m-1 < \alpha \le m, \ m \in \mathbb{N}.$$

The Caputo derivative of order $\alpha > 0$, along t, keeping x constant, is defined as

$$D_t^\alpha h(x,t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} \frac{\partial^m h(x,\tau)}{\partial \tau^m} d\tau, \ m-1 < \alpha \le m, \ m \in \mathbb{N}.$$

Let $h(x,t), \ g(x,t) \in L^1([0,a] \times [0,b]), a, b > 0, \ \theta = (\theta_1, \theta_2) \in (0,1] \times (0,1], \ \lambda$, $\gamma \in \mathbb{R}$, $\nu, \mu > -1$.
Then the following properties are attained.

(i) $D^\theta x^\nu t^\mu = \dfrac{\Gamma(\nu+1)\Gamma(\mu+1)}{\Gamma(1+\nu-\theta_1)\Gamma(1+\mu-\theta_2)} x^{\nu-\theta_1} t^{\mu-\theta_2}$.

(ii) $D^\theta k = 0$, *where k is a constant.*

(iii) $D^\theta(\lambda h(x,t) + \gamma g(x,t)) = \lambda \left(D^\theta h(x,t)\right) + \gamma \left(D^\theta g(x,t)\right)$.

(iv) $D_t^\alpha (J_t^\alpha h(x,t)) = h(x,t), \ m-1 < \alpha \le m, \ m \in \mathbb{N}$.

(v) $J_t^\alpha (D_t^\alpha h(x,t)) = h(x,t) - \displaystyle\sum_{k=0}^{m-1} h^k(x, 0^+) \frac{t^k}{k!}, \ m-1 < \alpha \le m,$

where $m \in \mathbb{N}$, $t > 0$ and $h^k(x, 0^+) := \lim_{t \to 0^+} \frac{\partial^k h(x,t)}{\partial t^k}$, $k = 0, 1, 2, \ldots, m-1$.

3. Haar wavelets and function approximation

In this section, Haar wavelets and fractional integration matrix of Haar wavelets are discussed.

3.1. **Haar wavelets.** The family of Haar wavelets for $t \in [0,1)$ is defined as

$$\psi_i(t) = \frac{2^{k/2}}{\sqrt{2^{J+1}}} \begin{cases} 1, \; for \; t \in [\frac{m}{2^k}, \frac{m+0.5}{2^k}) \\ -1, \; for \; t \in [\frac{m+0.5}{2^k}, \frac{m+1}{2^k}) \\ 0, \; otherwise. \end{cases} \tag{1}$$

where the integers $2^k, k = 0, 1, 2, ..., J,\; J \in \mathbb{N}$, denote the level of wavelets and the integers $m = 0, 1, 2, \ldots, 2^k - 1$, are the translation parameters. The index 'i' in (1) is evaluated using $i = 2^k + m + 1$. The minimal value of i is 2 and the maximal value of i is $P = 2^{J+1}$.
The scaling function $\psi_1(t)$ for the family of Haar wavelets is defined as

$$\psi_1(t) = \begin{cases} \frac{1}{\sqrt{2^{J+1}}}, \; for \; t \in [0,1) \\ 0, \; otherwise. \end{cases}$$

3.2. **Approximation of square integrable function.** An arbitrary function $f(x,t) \in L^2\left([0,1) \times [0,1)\right)$ can be written in terms of Haar wavelets as

$$f(x,t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} \psi_i(x) \psi_j(t),$$

where the Haar wavelet coefficients a_{ij}'s are resolved by the inner product $\langle \psi_i(x), \langle f, \psi_j(t) \rangle \rangle$. If $f(x,t)$ is made as piecewise constants approximately on each subinterval, then the infinite series can be cut off at finite terms, that is,

$$f(x,t) \approx \sum_{i=1}^{P} \sum_{j=1}^{P} a_{ij} \psi_i(x) \psi_j(t). \tag{2}$$

To find the numerical approximation of the function $f(x,t)$, we use the collocation points (x_i, t_j), where $i, j = 1, 2, 3, ..., P$. Discretizing (2) at the collocation points, we obtain

$$Y = H^T C H,$$

where $C = [a_{ij}]_{P \times P}$, $Y = [f(x_i, t_j)]_{P \times P}$ and $H = [\psi_i(x_j)]_{P \times P}$.
For instance, if $J = 2$, then we have

$$H = \begin{pmatrix} 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\ 0.3536 & 0.3536 & 0.3536 & 0.3536 & -0.3536 & -0.3536 & -0.3536 & -0.3536 \\ 0.5000 & 0.5000 & -0.5000 & -0.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5000 & 0.5000 & -0.5000 & -0.5000 \\ 0.7071 & -0.7071 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7071 & -0.7071 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7071 & -0.7071 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.7071 & -0.7071 \end{pmatrix}$$

3.3. **Matrix for fractional integration of Haar wavelets.** The fundamental idea of finding the fractional integration matrix of Haar wavelets[22] is here explored.
Let $I_i^\alpha(t) = J^\alpha\left(\psi_i(t)\right)$ and $r_i^\alpha(t) = I_i^\alpha(1)$. Then the integral $I_i^\alpha(t)$ can be evaluated

using (1) and is given by

$$I_i^{\alpha}(t) = \frac{2^{k/2}}{\sqrt{2^{J+1}}} \begin{cases} \varphi_1, \ for \ t \in [\frac{m}{2^k}, \frac{m+0.5}{2^k}) \\ \varphi_2, \ for \ t \in [\frac{m+0.5}{2^k}, \frac{m+1}{2^k}) \\ \varphi_3, \ for \ t \in [\frac{m+1}{2^k}, 1) \\ 0, \ otherwise. \end{cases} \tag{3}$$

where $\varphi_1 = \frac{1}{\Gamma(\alpha+1)}\left(t - \frac{m}{2^k}\right)^{\alpha}$,

$$\varphi_2 = \frac{1}{\Gamma(\alpha+1)}\left\{\left(t - \frac{m}{2^k}\right)^{\alpha} - 2\left(t - \frac{m+0.5}{2^k}\right)^{\alpha}\right\},$$

$$\varphi_3 = \frac{1}{\Gamma(\alpha+1)}\left\{\left(t - \frac{m}{2^k}\right)^{\alpha} - 2\left(t - \frac{m+0.5}{2^k}\right)^{\alpha} + \left(t - \frac{m+1}{2^k}\right)^{\alpha}\right\}.$$

If $J = 2$ and $\alpha = 0.5$, then the matrix $I^{0.5}$ of the integration (3) of order 0.5 is given by,

$$I^{0.5} = \begin{pmatrix} 0.0997 & 0.1728 & 0.2230 & 0.2639 & 0.2992 & 0.3308 & 0.3596 & 0.3863 \\ 0.0997 & 0.1728 & 0.2230 & 0.2639 & 0.0997 & -0.0147 & -0.0864 & -0.1415 \\ 0.1411 & 0.2443 & 0.0333 & -0.1154 & -0.0666 & -0.0343 & -0.0223 & -0.0162 \\ 0 & 0 & 0 & 0 & 0.1411 & 0.2443 & 0.0333 & -0.1154 \\ 0.1995 & -0.0534 & -0.0455 & -0.0188 & -0.0111 & -0.0075 & -0.0055 & -0.0043 \\ 0 & 0 & 0.1995 & -0.0534 & -0.0455 & -0.0188 & -0.0111 & -0.0075 \\ 0 & 0 & 0 & 0 & 0.1995 & -0.0534 & -0.0455 & -0.0188 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1995 & -0.0534 \end{pmatrix}$$

4. The proposed numerical technique for fractional Cahn-Allen equations

In this part, the Haar wavelet-based numerical technique for finding numerical solutions of fractional Cahn-Allen equations is expounded.
Let us consider the general fractional Cahn-Allen equations

$$D_t^{\alpha} h(x,t) - h_{xx}(x,t) - h(x,t) + h^3(x,t) = \lambda(x,t), \ 0 \le x, t \le 1, \ 0 < \alpha \le 1, \tag{4}$$

subject to the initial condition

$$h(x,0) = f(x) \tag{5}$$

and the boundary conditions

$$h(0,t) = g_0(t), \ h(1,t) = g_1(t), \tag{6}$$

where $f(x)$, $g_0(t)$, $g_1(t)$, $\lambda(x,t)$ are known functions, D_t^{α} is Caputo fractional differential operator with respect to 't', $h_{xx} = \frac{\partial^2 h}{\partial x^2}$, $h(x,t)$ is an unknown function.
Let

$$D_t^{\alpha} h_{xx}(x,t) = \sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij}\psi_i(x)\psi_j(t), \tag{7}$$

where a_{ij}'s are Haar coefficients to be resolved.
Now integrating (7), α times with regard to 't' from 0 to t, we get

$$h_{xx}(x,t) = \sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij}\psi_i(x)I_j^{\alpha}(t) + h_{xx}(x,0). \tag{8}$$

Also integrating (8) twice with regard to x from 0 to x, we attain

$$h(x,t)=\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}I_i^2(x)I_j^{\alpha}(t)+h(x,0)+h(0,t)-h(0,0)+x\left(h_x(0,t)-h_x(0,0)\right). \tag{9}$$

Putting $x=1$ in (9) and using (6), we attain

$$g_1(t)=\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}r_i^2(x)I_j^{\alpha}(t)+g_1(0)+g_0(t)-g_0(0)+h_x(0,t)-h_x(0,0),$$

which implies

$$h_x(0,t)-h_x(0,0)=g_1(t)-\left\{\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}r_i^2(x)I_j^{\alpha}(t)+g_1(0)+g_0(t)-g_0(0)\right\}. \tag{10}$$

Using (10) in (9), we attain

$$\begin{aligned}h(x,t)=&\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}I_i^2(x)I_j^{\alpha}(t)+f(x)+g_0(t)-g_0(0)\\&+x\left\{g_1(t)-\left(\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}r_i^2(x)I_j^{\alpha}(t)+g_1(0)+g_0(t)-g_0(0)\right)\right\}.\end{aligned} \tag{11}$$

Differentiating (11), α times with regard to 't', we get

$$\begin{aligned}D_t^{\alpha}h(x,t)=&\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}I_i^2(x)\psi_j(t)-x\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}r_i^2(x)\psi_j(t)\\&+D_t^{\alpha}\left[g_0(t)+x\Big(g_1(t)-g_0(t)\Big)\right].\end{aligned} \tag{12}$$

Using (8), (11) and (12) in (4), we attain

$$\begin{aligned}&\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}I_i^2(x)\psi_j(t)-x\sum_{i=1}^{P}\sum_{j=1}^{p}a_{ij}r_i^2(x)\psi_j(t)+D_t^{\alpha}\left[g_0(t)+x\Big(g_1(t)-g_0(t)\Big)\right]\\&-\left\{\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}\psi_i(x)I_j^{\alpha}(t)+h_{xx}(x,0)\right\}-\left\{\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}I_i^2(x)I_j^{\alpha}(t)+f(x)+g_0(t)-g_0(0)\right.\\&\left.+x\Big(g_1(t)-\left\{\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}r_i^2(x)I_j^{\alpha}(t)+g_1(0)+g_0(t)-g_0(0)\right\}\Big)\right\}+\left\{\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}I_i^2(x)I_j^{\alpha}(t)\right.\\&\left.+f(x)+g_0(t)-g_0(0)+x\left(g_1(t)-\left\{\sum_{i=1}^{P}\sum_{j=1}^{P}a_{ij}r_i^2(x)I_j^{\alpha}(t)+g_1(0)+g_0(t)-g_0(0)\right\}\right)\right\}^3=\lambda(x,t).\end{aligned} \tag{13}$$

Solving the equation (13) at the collocation points (x_i,t_j), $i,j=1,2,\dots,P$, we attain the Haar wavelet coefficients a_{ij}'s. Using these a_{ij}'s in (11), we get the Haar

wavelet-based numerical solutions of the equation (4).
In particular, suppose

$$\lambda(x,t) = (\alpha x + x - \alpha - 1)tx\Gamma(\alpha+1) + (x^2-x)^3 t^{\alpha+1} - t^{\alpha+1}(2+x^2-x),$$
$$f(x) = 0,\ 0 \le x \le 1 \tag{14}$$

and

$$g_0(t) = 0,\ g_1(t) = 0,\ 0 \le t \le 1. \tag{15}$$

Then the exact solution of (4) is $h(x,t) = (x^2-x)t^{\alpha+1}$ [21].

TABLE 1. Absolute errors attained by the proposed numerical technique for $\alpha = 0.5,\ P = 4$.

x	t	The proposed technique	Exact	Absolute error
0.125	0.125	-0.005139824270813	-0.004833737762017	3.060865e-04
0.375	0.375	-0.054104128600268	-0.053821796106076	2.823324e-04
0.625	0.625	-0.116073253238227	-0.115806066656557	2.671865e-04
0.875	0.875	-0.089603435902945	-0.089522076148400	8.135975e-05

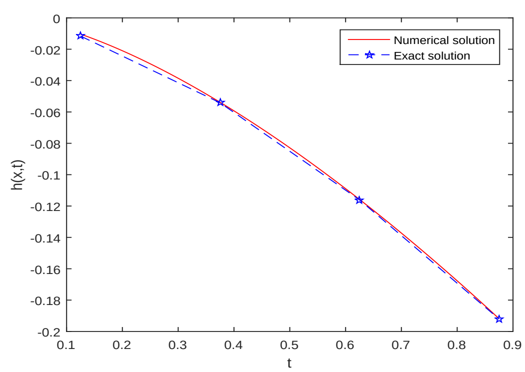


FIGURE 1. Comparison of numerical solutions attained by the proposed technique with the Exact solution for $P = 4$ at $x = 0.625$.

TABLE 2. Absolute errors attained by the proposed numerical technique for $\alpha = 0.5,\ P = 8$.

x	t	The proposed technique	Exact	Absolute error
0.0625	0.0625	-0.000959769646559	-0.000915527343750	4.424230e-05
0.1875	0.1875	-0.012427628352483	-0.012368771025730	5.885733e-05
0.3125	0.3125	-0.037639547262802	-0.037531658557888	1.078887e-04
0.4375	0.4375	-0.071330089682318	-0.071214375499236	1.157142e-04
0.5625	0.5625	-0.103923596668848	-0.103820800781250	1.027959e-04
0.6875	0.6875	-0.122546908425943	-0.122470580942689	7.632748e-05
0.8125	0.8125	-0.111617829217056	-0.111573150430923	4.467879e-05
0.9375	0.9375	-0.053201296456909	-0.053187332330119	1.396413e-05

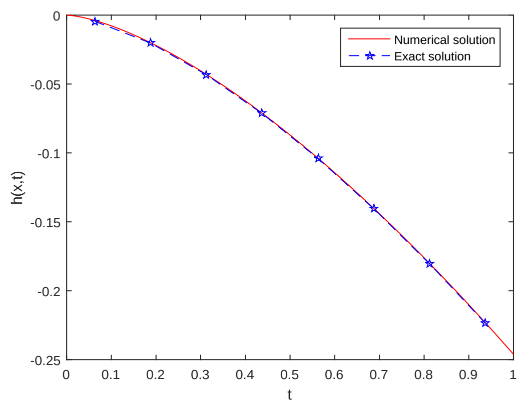
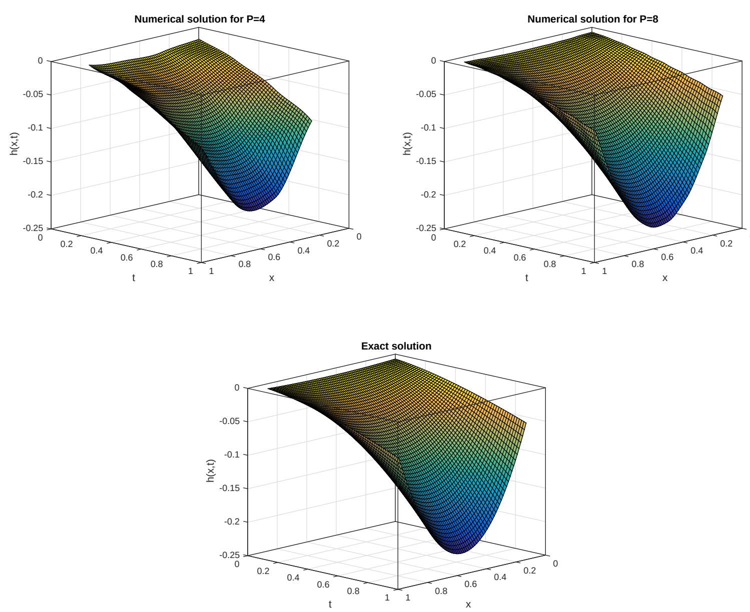


FIGURE 2. Comparison of numerical solutions attained by the proposed technique with the Exact solution for $P = 8$ at $x = 0.5625$.



Applying the initial condition and the boundary conditions in (13), we attain

$$\sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij} I_i^2(x)\psi_j(t) - x\sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij} r_i^2(x)\psi_j(t) - \left\{\sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij}\psi_i(x) I_j^{\alpha}(t)\right\}$$
$$- \left\{\sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij} I_i^2(x) I_j^{\alpha}(t) - x\sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij} r_i^2(x) I_j^{\alpha}(t)\right\}$$
$$+ \left\{\sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij} I_i^2(x) I_j^{\alpha}(t) - x\sum_{i=1}^{P}\sum_{j=1}^{P} a_{ij} r_i^2(x) I_j^{\alpha}(t)\right\}^3 = \lambda(x,t). \quad (16)$$

Eqn.(16) can also be written in matrix form as

$$\left(I^2\right)^T CH - X\left(R^2\right)^T CH - H^T CI^\alpha - \left(\left(I^2\right)^T CI^\alpha - X\left(R^2\right)^T CI^\alpha\right) + \left(\left(I^2\right)^T CI^\alpha - X\left(R^2\right)^T CI^\alpha\right)^3 = \Lambda, \quad (17)$$

where $I^2 = [I_i^2(x_j)]_{P\times P}$, $I^\alpha = [I_i^\alpha(x_j)]_{P\times P}$, $R^2 = [r_i^2(x_j)]_{P\times P}$, $X = diag(x_1, x_2, ..., x_P)$, $\Lambda = [\lambda(x_i, t_j)]_{P\times P}$. Solving the system (17) at the collocation points (x_i, t_j), $i, j = 1, 2, \ldots, P$, we get the Haar wavelet coefficient a_{ij}'s. Using these values a_{ij}'s in (11), we get the Haar wavelet-based numerical solutions of the equation (4). The absolute errors incurred by the proposed numerical strategy for $P = 4$ and $P = 8$ respectively are exhibited in table 1 and 2. As the absolute errors become smaller with P increasing in table 1 and 2, we find that the proposed strategy can reach a higher degree of precision. This affirms that the numerical outcomes accomplished by the proposed strategy are in acceptable concurrence with the exact solution.

5. Conclusion

The proposed technique is successfully employed to attain the numerical solutions of fractional Cahn-Allen equations. The comparison of exact solutions with the numerical solutions obtained by the proposed technique and the graphical illustrations show that the proposed technique is very effective, fast and flexible for fractional Cahn-Allen equations.

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