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FRACTIONAL CALCULUS OF GENERALIZED K- WRIGHT FUNCTION

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Abstract. In this paper, authors studied fractional calculus properties viz. Riemann-Liouville fractional integral and derivative of the generalized K- Wright function ${}_p\Psi_q^k(z)$. Family of Mittag-Leffler function introduced by several authors ([2], [3], [5], [6], [8], [9], [10], [11]) are particular cases of generalized K-Wright function.

1. Introduction

Generalized K-Gamma Function $\Gamma_k(x)$ defined as (Diaz and Pariguan [1])

$$\Gamma_k(x) = \lim_{n\to\infty} \frac{n!\ k^n (nk)^{\frac{x}{k}\ -\ 1}}{(x)_{n,k}}\ ,\ k>0,\ x \in C\backslash kZ^- \tag{1}$$

where $(x)_{n,k}$ is the k-Pochhammer symbol and is given by

$$(x)_{n,k} = x(x+k)(x+2k)\dots\dots\dots\dots(x+(n-1)k), \quad x\in C,\ k\in R,\ n\in N^+ \tag{2}$$

For $Re(x)>0$, then $\Gamma_k(x)$ defined as the integral

$$\Gamma_k(x) = \int_0^\infty t^{x-1} e^{-\ \frac{t^k}{k}} dt \tag{3}$$

this follows

$$\Gamma_k(x) = k^{\frac{x}{k}\ -1}\Gamma\left(\frac{x}{k}\right) \tag{4}$$

The generalized Wright function defined by (Wright [11])
For $z\in C;;\ a_i, b_j \in C$ and $\alpha_i, \beta_j \in R\ \ (\alpha_i, \beta_j \neq 0;; i=1,2,\dots,p;; j=1,2,\dots,q)$ as

$$ {}_p\Psi_q(z) = {}_p\Psi_q\left[\begin{array}{c|c} (a_i,\alpha_i)_{1,p} & \\ (b_j,\beta_j)_{1,q} & z\end{array}\right] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)} \frac{z^n}{n!} \tag{5}$$

The left- and right-hand sided fractional integral operators are defined for $\alpha > 0\ and\ a=0$ as (Samko et al [8])

$$\left(I_{0+}^{\alpha} f\right)(x) = \frac{1}{\Gamma(\alpha)}\int_0^x \frac{f(t)}{(x-t)^{1-\alpha}} dt;; \tag{6}$$

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and

$$\left(I_{-}^{\alpha}f\right)(x)=\frac{1}{\Gamma(\alpha)}\int_{x}^{\infty}\frac{f(t)}{(t-x)^{1-\alpha}}dt \tag{7}$$

and corresponding fractional differentiation operators defined as

$$\begin{aligned}\left(D_{0+}^{\alpha}f\right)(x) &= \left(\frac{d}{dx}\right)^{[Re(\alpha)]+1}\left(I_{0+}^{1-\alpha+[Re(\alpha)]}f\right)(x)\\ &= \frac{1}{\Gamma(1-\alpha+[Re(\alpha)])}\left(\frac{d}{dx}\right)^{[Re(\alpha)]+1}\int_{0}^{x}\frac{f(t)}{(x-t)^{\alpha-[Re(\alpha)]}}dt\end{aligned} \tag{8}$$

and

$$\begin{aligned}\left(D_{-}^{\alpha}f\right)(x) &= \left(-\frac{d}{dx}\right)^{[Re(\alpha)]+1}\left(I_{-}^{1-\alpha+[Re(\alpha)]}f\right)(x)\\ &= \frac{1}{\Gamma(1-\alpha+[Re(\alpha)])}\left(-\frac{d}{dx}\right)^{[Re(\alpha)]+1}\int_{x}^{\infty}\frac{f(t)}{(t-x)^{\alpha-[Re(\alpha)]}}dt\end{aligned} \tag{9}$$

and
The next assertion is well known (Samko et al [8])
For $\alpha\in C\ \ (Re(\alpha)>0)\ \ and\ \gamma\in C;;$
If $Re(\gamma)>0$, then $\left(I_{0+}^{\alpha}t^{\gamma-1}\right)(x)=\frac{\Gamma(\gamma)}{\Gamma(\alpha+\gamma)}x^{\alpha+\gamma-1}$ (10)
If $Re(\gamma)>Re(\alpha)>0$, then $\left(I_{-}^{\alpha}t^{-\ \gamma}\right)(x)=\frac{\Gamma(\gamma-\alpha)}{\Gamma(\gamma)}x^{\alpha-\gamma}$ (11)

2. GENERALIZED K- WRIGHT FUNCTION

The Concept of the Generalized K-Wright function introduced by Gehlot and Prajapati [4] as
Definition: The Generalized K- Wright Function, ${}_{p}\psi_{q}^{k}(z)$, defined for $k\in R^{+};;$ $z\in C,\ a_i,b_j\in C,\ \alpha_i,\beta_j\in R\ (\alpha_i,\beta_j\neq 0;;i=1,2,\ldots,p;;j=1,2,\ldots,q)$ *and* $(a_i+\alpha_i n),\ (b_j+\beta_j n)\ \in C\backslash kZ^{-}$

$${}_{p}\psi_{q}^{k}(z)={}_{p}\psi_{q}^{k}\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta j)_{1,q}\end{array}\middle|z\right]=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)}\frac{z^n}{n!} \tag{12}$$

For convergence, we use the following notations

$$\Delta=\sum_{j=1}^{q}\left(\frac{\beta_j}{k}\right)-\sum_{j=1}^{p}\left(\frac{\alpha_i}{k}\right);\delta=\prod_{i=1}^{p}\left|\frac{\alpha_i}{k}\right|^{-\ \frac{\alpha_i}{k}}\prod_{j=1}^{q}\left|\frac{\beta_j}{k}\right|^{\frac{\beta_j}{k}};;\ \mu=\sum_{j=1}^{q}\left(\frac{b_j}{k}\right)-\sum_{j=1}^{p}\left(\frac{a_i}{k}\right)+\frac{p-q}{2}$$

Theorem 1. For $k\in R^{+};;\ z\in C;;\ \ a_i,b_j\in C;;\ \ \alpha_i,\beta_j\in R\ (\alpha_i,\beta_j\neq 0;;i=1,2,\ldots,p;;j=1,2,\ldots,q)$ *and* $(a_i+\alpha_i n),\ (b_j+\beta_j n)\ \in C\backslash kZ^{-}$

(1) If $\Delta\ >-1$ then series (10) is absolutely convergent for all $z\in C$ and Generalized K- Wright Function ${}_{p}\Psi_{q}^{k}(z)$ is an entire function of z.

(2) If $\Delta = -1$, then series (10) is absolutely convergent for all $|z| < \delta$ and of

$$|z| = \delta,\ Re\,(\mu) > 1/2$$

3. SPECIAL CASES

For the appropriate values of the parameters p, q, k, r, γ *and* β , some interesting special cases of Generalized K- Wright Function ${}_p\Psi_q^k\,(z)$ can be obtained in terms of Mittag-Leffler function defined as references [2], [3], [5], [6], [8], [9], [10], [11].

4. FRACTIONAL INTEGRATION OF THE GENERALIZED K-WRIGHT FUNCTION

In this section, author establishes fractional integration of the generalized K-Wright function.

Theorem 2. Let $\alpha, \gamma \in C$ such that $Re\,(\alpha) > 0$, $Re\,(\gamma) > 0$; $a \in C,\ \mu > 0$ then for $\Delta > -1$ fractional Integration I_{0+}^{α} of generalized K-Wright function ${}_p\Psi_q^k\,(z)$ is given by

$$\left(I_{0+}^{\alpha}\left(t^{\frac{\gamma}{k}-1}{}_p\Psi_q^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\middle|\,at^{\frac{\mu}{k}}\right]\right)\right)(x)$$

$$= k^{\alpha}x^{\frac{\gamma}{k}+\alpha-1}{}_{p+1}\Psi_{q+1}^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\,,(\gamma,\mu)\\(b_j,\beta_j)_{1,q}\,,(\gamma+\alpha k,\mu)\end{array}\middle|\,ax^{\frac{\mu}{k}}\right] \tag{13}$$

Proof.

According to theorem 1, the generalized K-Wright functions in both side of (13) exist for $x > 0$. Consider the right- hand side of (13) and using the definition (12), we have

$$A \equiv \left(I_{0+}^{\alpha}\left(t^{\frac{\gamma}{k}-1}{}_p\Psi_q^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\middle|\,at^{\frac{\mu}{k}}\right]\right)\right)(x)$$

this can be written as,

$$A \equiv \left(I_{0+}^{\alpha}\left(t^{\frac{\gamma}{k}-1}\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k\,(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k\,(b_j+\beta_j n)}\frac{\left(at^{\frac{\mu}{k}}\right)^n}{n!}\right)\right)(x)$$

using term-by term integration of the series in the right –hand side of above equation and using (10), we obtain

$$A = \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k\,(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k\,(b_j+\beta_j n)}\frac{(a)^n}{n!}\left(I_{0+}^{\alpha}\left(t^{\frac{\gamma}{k}+\frac{\mu n}{k}-1}\right)\right)(x)$$

this reduces to,

$$A = \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k\,(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma_k\,(b_j+\beta_j n)}\frac{(a)^n}{n!}\frac{\Gamma\left(\frac{\gamma+\mu n}{k}\right)}{\Gamma\left(\frac{\gamma+\mu n}{k}+\alpha\right)}x^{\frac{\gamma+\mu n}{k}+\alpha-1}$$

using the relation (4), we have

$$A = k^{\alpha}x^{\frac{\gamma}{k}+\alpha-1}{}_{p+1}\Psi_{q+1}^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\,,(\gamma,\mu)\\(b_j,\beta_j)_{1,q}\,,(\gamma+\alpha k,\mu)\end{array}\middle|\,ax^{\frac{\mu}{k}}\right]$$

Theorem 3. Let $\alpha, \gamma \in C$ such that $Re(\alpha) > 0$, $Re(\gamma) > 0$; $a \in C$, $\mu > 0$ then for $\Delta > -1$ the fractional Integration I_{-}^{α} of generalized K-Wright function ${}_{p}\Psi_{q}^{k}(z)$ is given by

$$\left(I_{-}^{\alpha} \left(t^{-\frac{\gamma}{k}} {}_{p}\Psi_{q}^{k} \left[\begin{array}{c} (a_i, \alpha_i)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{array} \middle| at^{-\frac{\mu}{k}} \right] \right) \right)(x)$$

$$= k^{\alpha} x^{\alpha - \frac{\gamma}{k}} {}_{p+1}\Psi_{q+1}^{k} \left[\begin{array}{c} (a_i, \alpha_i)_{1,p}, (\gamma - \alpha k, \mu) \\ (b_j, \beta_j)_{1,q}, (\gamma, \mu) \end{array} \middle| ax^{-\frac{\mu}{k}} \right] \tag{14}$$

Proof.
According to theorem 1, the generalized K-Wright functions in both side of (14) exist for $x > 0$. Consider the right hand side of (14) and using the definition (12),

$$B \equiv \left(I_{-}^{\alpha} \left(t^{-\frac{\gamma}{k}} {}_{p}\Psi_{q}^{k} \left[\begin{array}{c} (a_i, \alpha_i)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{array} \middle| at^{-\frac{\mu}{k}} \right] \right) \right)(x)$$

this can be written as,

$$B = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k (a_i + \alpha_i n)}{\prod_{j=1}^{q} \Gamma_k (b_j + \beta_j n)} \frac{a^n}{n!} \left(I_{-}^{\alpha} \left(t^{-\frac{\gamma + \mu n}{k}} \right) \right)(x)$$

using (11), we have

$$B = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma_k (a_i + \alpha_i n)}{\prod_{j=1}^{q} \Gamma_k (b_j + \beta_j n)} \frac{a^n}{n!} \frac{\Gamma\left(\frac{\gamma + \mu n}{k} - \alpha\right)}{\Gamma\left(\frac{\gamma + \mu n}{k}\right)} x^{\alpha - \frac{\gamma + \mu n}{k}}$$

using relation (4), we finally arrive at

$$B \equiv k^{\alpha} x^{\alpha - \frac{\gamma}{k}} {}_{p+1}\Psi_{q+1}^{k} \left[\begin{array}{c} (a_i, \alpha_i)_{1,p}, (\gamma - \alpha k, \mu) \\ (b_j, \beta_j)_{1,q}, (\gamma, \mu) \end{array} \middle| ax^{-\frac{\mu}{k}} \right]$$

5. FRACTIONAL DIFFERENTIATION OF THE GENERALIZED K-WRIGHT FUNCTION

This section deals with fractional differentiation of the generalized K-Wright function (12).

Theorem 4. Let $\alpha, \gamma \in C$ such that $Re(\alpha) > 0$, $Re(\gamma) > 0;;$ $a \in C$, $\mu > 0$ then for $\Delta > -1$ the fractional differentiation D_{0+}^{α} of generalized K-Wright function ${}_{p}\Psi_{q}^{k}(z)$ is given by,

$$\left(D_{0+}^{\alpha} \left(t^{\frac{\gamma}{k} - 1} {}_{p}\Psi_{q}^{k} \left[(a_i, \alpha_i)_{1,p} (b_j, \beta_j)_{1,q} \middle| at^{\frac{\mu}{k}} \right] \right) \right)(x)$$

$$= k^{-\alpha} x^{\frac{\gamma}{k} - \alpha - 1} {}_{p+1}\Psi_{q+1}^{k} \left[\begin{array}{c} (a_i, \alpha_i)_{1,p}, (\gamma, \mu) \\ (b_j, \beta_j)_{1,q}, (\gamma - \alpha k, \mu) \end{array} \middle| ax^{\frac{\mu}{k}} \right] \tag{15}$$

Proof.
According to theorem 1, the generalized K-Wright functions in both side of (15) exist for $x > 0$.
Let $r = [Re(\alpha)] + 1$ then using (8), in right side of (15), we have,

$$E = \left(D_{0+}^{\alpha} \left(t^{\frac{\gamma}{k} - 1} {}_{p}\Psi_{q}^{k} \left[\begin{array}{c} (a_i, \alpha_i)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{array} \middle| at^{\frac{\mu}{k}} \right] \right) \right)(x)$$

this yields,

$$E = \left(\frac{d}{dx}\right)^r \left(I_{0+}^{r-\alpha}\left(t^{\frac{\gamma}{k}-1}{}_p\Psi_q^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\middle| at^{\frac{\mu}{k}}\right]\right)\right)(x)$$

using result (13), we obtain

$$E = \left(\frac{d}{dx}\right)^r \left(x^{\frac{\gamma}{k}+r-\alpha-1}k^{r-\alpha}{}_{p+1}\Psi_{q+1}^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p},(\gamma,\mu)\\(b_j,\beta_j)_{1,q},(\gamma+(r-\alpha)k,\mu)\end{array}\middle| ax^{\frac{\mu}{k}}\right]\right)$$

using (12), above equation reduces to,

$$E = \sum_{n=0}^{\infty}\frac{k^{r-\alpha}\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)\Gamma_k(\gamma+\mu n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\Gamma_k(\gamma+(r-\alpha)k+\mu n)}\frac{(a)^n}{n!}\left(\frac{d}{dx}\right)^r\left(x^{\frac{\gamma}{k}+\frac{\mu n}{k}+r-\alpha-1}\right)$$

this gives,

$$E = k^{-\alpha}\,x^{\frac{\gamma}{k}-\alpha-1}{}_{p+1}\Psi_{q+1}^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p},(\gamma,\mu)\\(b_j,\beta_j)_{1,q},(\gamma-\alpha k,\mu)\end{array}\middle| ax^{\frac{\mu}{k}}\right]$$

Theorem 5. Let $\alpha,\gamma\in C$ such that $Re(\alpha)>0$, $Re(\gamma)>[Re(\alpha)]+1-Re(\alpha)$; $a\in C$, $\mu>0$ then for $\Delta>-1$, the fractional differentiation D_-^α of generalized K-Wright function ${}_p\Psi_q^k(z)$ is given by

$$\left(D_-^\alpha\left(t^{-\frac{\gamma}{k}}{}_p\Psi_q^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\middle| at^{-\frac{\mu}{k}}\right]\right)\right)(x)$$

$$= k^{-\alpha}x^{-\alpha-\frac{\gamma}{k}}{}_{p+1}\Psi_{q+1}^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p},(\gamma+\alpha k,\mu)\\(b_j,\beta_j)_{1,q},(\gamma,\mu)\end{array}\middle| ax^{-\frac{\mu}{k}}\right] \tag{16}$$

Proof.
According to theorem 1, the generalized K-Wright functions in both side of (16) exist for $x>0$.
Let $r=[Re(\alpha)]+1$, where $[Re(\alpha)]$ is an integral part of $Re(\alpha)$.
Using (9), the right side of (16) gives

$$F = \left(D_-^\alpha\left(t^{-\frac{\gamma}{k}}{}_p\Psi_q^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\middle| at^{-\frac{\mu}{k}}\right]\right)\right)(x)$$

yields,

$$F = \left(-\frac{d}{dx}\right)^r\left(I_-^{r-\alpha}\left(t^{-\frac{\gamma}{k}}{}_p\Psi_q^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\middle| at^{-\frac{\mu}{k}}\right]\right)\right)(x)$$

using (14), we get

$$F = \left(-\frac{d}{dx}\right)^r\left(x^{r-\alpha-\frac{\gamma}{k}}\quad k^{r-\alpha}\,{}_{p+1}\Psi_{q+1}^k\left[\begin{array}{c}(a_i,\alpha_i)_{1,p},(\gamma-(r-\alpha)k,\mu)\\(b_j,\beta_j)_{1,q},(\gamma,\mu)\end{array}\middle| ax^{-\frac{\mu}{k}}\right]\right)$$

using (12), this immediately leads to,

$$F = \sum_{n=0}^{\infty}\frac{k^{r-\alpha}\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)\Gamma_k(\gamma-(r-\alpha)k+\mu n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\;\Gamma_k(\gamma+\mu n)}\frac{(a)^n}{n!}$$
$$\times\left(-\frac{d}{dx}\right)^r\left(x^{r-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}}\right)$$

on simplifying above equation, we get

$$F = \sum_{n=0}^{\infty}\frac{k^{r-\alpha}\prod_{i=1}^{p}\Gamma_k(a_i+\alpha_i n)\Gamma_k(\gamma-(r-\alpha)k+\mu n)}{\prod_{j=1}^{q}\Gamma_k(b_j+\beta_j n)\;\Gamma_k(\gamma+\mu n)}\frac{(a)^n}{n!}$$
$$\times\frac{(-1)^r\Gamma\left(1+r-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}\right)}{\Gamma\left(1-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}\right)}\left(x^{-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}}\right)$$

using (4), above equation can be written as,

$$F=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k\left(a_i+\alpha_i n\right)}{\prod_{j=1}^{q}\Gamma_k\left(b_j+\beta_j n\right)\,\Gamma\left(\frac{\gamma+\mu n}{k}\right)}\frac{(a)^n}{n!}$$

$$\times\frac{(-1)^r\Gamma\left(\alpha-r+\frac{\gamma}{k}+\frac{\mu n}{k}\right)\Gamma\left(1+r-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}\right)}{\Gamma\left(1-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}\right)}\left(x^{-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}}\right) \tag{17}$$

using reflection formula for the gamma-function, (Samko et al [8]),

$$\frac{1}{\Gamma\left(1-\alpha-\frac{\gamma}{k}-\frac{\mu n}{k}\right)}=\frac{\Gamma\left(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k}\right)\sin\left[\left(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k}\right)\pi\right]}{\pi} \tag{18}$$

and

$$\Gamma\left(\alpha-r+\frac{\gamma}{k}+\frac{\mu n}{k}\right)\Gamma\left(1-\left(\alpha-r+\frac{\gamma}{k}+\frac{\mu n}{k}\right)\right)$$

$$=\frac{\pi}{\sin\left[\left(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k}\right)\pi-r\pi\right]}$$

$$=\frac{\pi}{\sin\left[\left(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k}\right)\pi\right]\cos\left(r\pi\right)}$$

$$=\frac{\pi(-1)^r}{\sin\left[\left(\alpha+\frac{\gamma}{k}+\frac{\mu n}{k}\right)\pi\right]}$$

substituting values of (18) and (19) in (17), we finally arrived at

$$F=k^{-\alpha}x^{-\alpha-\frac{\gamma}{k}}\,{}_{p+1}\Psi_{q+1}^{k}\left[\begin{array}{c}\left(a_i,\alpha_i\right)_{1,p},\left(\gamma+\alpha k,\mu\right)\\\left(b_j,\beta_j\right)_{1,q},\left(\gamma,\mu\right)\end{array}\middle|\,ax^{-\frac{\mu}{k}}\right]$$

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