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NUMERICAL SOLUTION OF SYSTEM OF FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS BY DISCRETE ADOMIAN DECOMPOSITION METHOD

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Abstract. The aim of this paper is to obtain the solution of linear as well as nonlinear system of fractional partial differential equations with initial conditions, using space discrete Adomian decomposition method. It is verified by comparing with exact solution when α=1. Solutions of numerical examples are graphically represented by using MATLAB software.

1. Introduction

Partial differential equations arise in every field of science and technology, in particular in physics, chemistry, biology,engineering and bioengineering. System of partial differential equations have attracted much attention in studying evolution equations describing wave propagation, in investigating the shallow water waves [14, 15, 24] and in examining the chemical reaction-diffusion model of Brusselator[3].Fractional differential equations are increasingly used to model problems in acoustics and thermal systems, rheology and mechanical systems, signal processing and systems identification, control and robotics and other areas of applications (see [5, 25]). The interdisciplinary applications show the importance and necessity of fractional calculus. It motivates us to construct a variety of efficient methods for fractional differential equations such as integral transform method[18, 19],new iterative method[10, 12, 13] and Adomian decomposition method [4, 11]. Adomian decomposition method(ADM) [1, 2] and references theirin, has proved to be a very useful tool while dealing with nonlinear equations. In the last two decades, extensive work has been done using ADM[9, 17, 22, 23]. It provides approximate solutions for nonlinear equations without linearization & perturbation. Shawagfeh[20], Li et al. [8] has employed ADM for solving nonlinear fractional differential equations.Recently, considerable attention has been given to ADM for solving nonlinear fractional partial differential equations. The discrete ADM was first used to obtain the numerical solutions of the discrete nonlinear Schrodinger equation[6].

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We organize the paper as follows. In section 2, we define preliminary definitions and some properties of Riemann-Liouville(R-L)integral and relation between R-L integral and Caputo fractional derivative. Section 3, is devoted for analysis of discrete ADM. In section 4, we illustrate the method solving linear as well as nonlinear system of fractional partial differential equations with suitable initial conditions.

2. Preliminaries and notations

In this section, we set up notation and basic definitions and main properties of R-L integral and relation between R-L integral and Caputo fractional derivative from fractional calculus.

Definition 2.1. [16] *A real function $f(x), x > 0$ is said to be in space $C_\alpha, \alpha \in \Re$ if there exists a real number $p > \alpha$ such that $f(x) = x^p f_1(x)$ where $f_1(x) \in C[0, \infty)$.*

Definition 2.2. [16] *A function $f(x), x > 0$ is said to be in space $C_\alpha^m, m \in N \bigcup \{0\}$ if $f^m \in C_\alpha$.*

Definition 2.3. [18] *Let $f \in C_\alpha$ and $\alpha \geq -1$, then Riemann-Liouville fractional integral of $f(x,t)$ of order α is denoted by $J^\alpha f(x,t)$ and is difined as*

$$J^\alpha f(x,t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(x,\tau) d\tau, t > 0, \alpha > 0$$

The well known property of the Riemann-Liouville operatorJ^α is

$$J^\alpha t^\gamma = \frac{\Gamma(\gamma+1) t^{\gamma+\alpha}}{\Gamma(\gamma+\alpha+1)}$$

Definition 2.4. [7] *For m to be the smallest integer that exceeds $\alpha > 0$, the Caputo fractional derivative of $u(x,t)$ of order $\alpha > 0$ is defined as*

$$D_t^\alpha u(x,t) = \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} \frac{\partial^m u}{\partial t^m} d\tau, & \text{for } m-1 < \alpha < m; \\ \frac{\partial^m u(x,t)}{\partial t^m}, & \text{for } \alpha = m \in N. \end{cases}$$

Note that the relation between Riemann-Liouville operator and Caputo fractional differential operator is given as follows.

$$J^\alpha (D^\alpha f(x,t)) = (J^{m-\alpha} f^{(m)})(t) = f(x,t) - \sum_{k=0}^{m-1} f^{(k)}(0) \frac{t^k}{k!}$$

3. Discrete Adomian Decomposition Method

Consider the time fractional system of partial differential equations of order α,

$$\begin{aligned} D_t^\alpha u + N_1(u,v,u_x,v_x) &= g_1(x,t) \\ D_t^\alpha v + N_2(u,v,u_x,v_x) &= g_2(x,t) \end{aligned} \quad 0 < \alpha \leq 1 \tag{1}$$

with the initial conditions

$$u(x,0) = f_1(x); \quad v(x,0) = f_2(x) \tag{2}$$

where $D_t^\alpha(.)$ is a Caputo fractional derivative of order $\alpha(0 < \alpha \leq 1)$, $N_1 \& N_2$ are nonlinear operators, $g_1 \& g_2$ are inhomogeneous functions. The discrete form of equation (1)-(2) is as follows

$$\begin{aligned} D_t^\alpha u_j(t) + N_1\Big(u_j(t), v_j(t), D_h u_j(t), D_h v_j(t)\Big) &= g_{1j}(t) \\ D_t^\alpha v_j(t) + N_2\Big(u_j(t), v_j(t), D_h u_j(t), D_h v_j(t)\Big) &= g_{2j}(t) \end{aligned} \tag{3}$$

with the initial conditions

$$u_j(0) = f_{1j}; \quad v_j(0) = f_{2j} \tag{4}$$

where $u(x,t) = u(j\Delta x, t)$ is the discrete function and it is denoted by $u_j(t)$,$\Delta x = h$ and $f_1(x, 0) = f_1(j\Delta x)$ is the discrete function & is denoted by f_{1j} and $f_2(x, 0) = f_2(j\Delta x)$ is the discrete function & denoted by f_{2j}. The standard central differences [21] $D_h u_j(t)$ and $D_h v_j(t)$ are defined by

$$D_h u_j(t) = \frac{u_{j+1}(t) - u_{j-1}(t)}{2h}, \quad D_h v_j(t) = \frac{v_{j+1}(t) - v_{j-1}(t)}{2h}$$

Applying the operator J^α to the system (3) and use the initial conditions (4), we get

$$\begin{aligned} u_j(t) &= f_{1j} + J^\alpha g_{1j}(t) - J^\alpha N_1\Big(u_j(t), v_j(t), D_h u_j(t), D_h v_j(t)\Big) \\ v_j(t) &= f_{2j} + J^\alpha g_{2j}(t) - J^\alpha N_2\Big(u_j(t), v_j(t), D_h u_j(t), D_h v_j(t)\Big) \end{aligned} \tag{5}$$

As per the Adomian decomposition method the linear terms $u_j(t)$, $v_j(t)$ and the nonlinear operators N_1 and N_2 should be decomposed by an infinite series of components such as

$$u_j(t) = \sum_{n=0}^{\infty} u_{jn}(t), \quad v_j(t) = \sum_{n=0}^{\infty} v_{jn}(t) \tag{6}$$

and

$$\begin{aligned} N_1\Big(u_j(t), v_j(t), D_h u_j(t), D_h v_j(t)\Big) &= \sum_{n=0}^{\infty} A_n \\ N_2\Big(u_j(t), v_j(t), D_h u_j(t), D_h v_j(t)\Big) &= \sum_{n=0}^{\infty} B_n \end{aligned} \tag{7}$$

respectively.Note that $u_{jn}(t), v_{jn}(t)$, $(n \geq 0)$ are the approximations of $u_j(t) \& v_j(t)$ and those will be elegantly determined also $A_n \& B_n$ $(n \geq 0)$ are Adomian polynomials those can be generated for all forms of nonlinearity. The Adomian polynomial $A_n \& B_n$ are generated according to nonlinearity.In general the Adomian polynomial is defined as

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} F\Big(\sum_{k=0}^{\infty} \lambda^k u_{jk}\Big)\right]_{\lambda=0}, \quad n \geq 0 \tag{8}$$

Substituting (7) and (6) into (5)gives

$$\begin{aligned} \sum_{n=0}^{\infty} u_{jn}(t) &= f_{1j} + J^{\alpha} g_{1j} - J^{\alpha}\Big(\sum_{n=0}^{\infty} A_n\Big) \\ \sum_{n=0}^{\infty} v_{jn}(t) &= f_{2j} + J^{\alpha} g_{2j} - J^{\alpha}\Big(\sum_{n=0}^{\infty} B_n\Big) \end{aligned} \tag{9}$$

On simplifying equations in (9), we get the following recursive relations as follow:

$$\begin{aligned} u_{j0}(t) &= f_{1j} + J^{\alpha} g_{1j}, \quad u_{jn+1}(t) = -J^{\alpha}(A_n); n \geq 0 \\ v_{j0}(t) &= f_{2j} + J^{\alpha} g_{2j}, \quad v_{jn+1}(t) = -J^{\alpha}(B_n); n \geq 0 \end{aligned} \tag{10}$$

We know the zeroth components from the initial conditions and using the above recurrence relations, we find the remaining components.

Remark 3.1. *The above discrete Adomian decomposition method discussed for the time fractional system of two partial differential equations can be extended to the time fractional system of any finite number of partial differential equations.*

4. Numerical Examples

In this section we solve system of fractional linear as well as nonlinear partial differential equations, with suitable initial conditions.

Example 4.1. *Consider the linear system of fractional partial differential equations*

$$\begin{aligned} D_t^{\alpha} u + u_x - 2v = 0 \\ D_t^{\alpha} v + v_x - 2u = 0 \end{aligned} \tag{11}$$

with initial conditions

$$u(x, 0) = sinx, \quad v(x, 0) = cosx \tag{12}$$

It is called initial value problem (IVP). The discrete form of IVP (11)-(12) is

$$\begin{aligned} D_t^{\alpha} u_j(t) + D_h u_j(t) - 2v_j(t) = 0 \\ D_t^{\alpha} v_j(t) + D_h v_j(t) - 2u_j(t) = 0 \end{aligned} \tag{13}$$

with initial conditions

$$u_{j0} = sinjh, \quad v_{j0} = cosjh \tag{14}$$

It is called discrete IVP.Operating the operator J^{α} on equations in (13)and using initial conditions,we obtain

$$\begin{aligned} u_j(t) &= sin(jh) + J^{\alpha}(2v_j(t) - D_h u_j(t)) \\ v_j(t) &= cos(jh) + J^{\alpha}(2u_j(t) - D_h v_j(t)) \end{aligned} \tag{15}$$

Using Adomian procedure we assumes that equations in (15) have series solution. We obtain the following recurrence relations

$$\begin{aligned} u_{j0} &= sin(jh), \quad u_{jn+1}(t) = J^{\alpha}(2v_{jn} - D_h u_{jn}(t)) \\ v_{j0} &= cos(jh), \quad v_{jn+1}(t) = J^{\alpha}(2u_{jn} - D_h v_{jn}(t)) \end{aligned} \tag{16}$$

Since u_{j0} and v_{j0} are known, from recurrence relations in (16), we find u_{j1}

$$\begin{aligned} u_{j1}(t) &= J^{\alpha}(2v_{j0} - D_h u_{j0}(t)) \\ u_{j1}(t) &= (2 - \frac{sin(h)}{h})cos(jh)\frac{t^{\alpha}}{\Gamma(\alpha+1)} \end{aligned}$$

and v_{j1}

$$v_{j1}(t) = J^{\alpha}(2u_{j0} - D_h v_{j0}(t))$$
$$v_{j1}(t) = -(2 - \frac{sin(h)}{h})sin(jh)\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

Similarly we find u_{j2} and v_{j2} using u_{j1} and v_{j1} in recurrence relations in (16)

$$u_{j2}(t) = J^{\alpha}(2v_{j1}(t) - D_h u_{j1}(t))$$
$$u_{j2}(t) = -(2 - \frac{sin(h)}{h})^2 sin(jh)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

and v_{j2}

$$v_{j2}(t) = J^{\alpha}(2u_{j1}(t) - D_h v_{j1}(t))$$
$$v_{j2}(t) = -(2 - \frac{sin(h)}{h})^2 cos(jh)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

Similarly we find u_{j3} and v_{j3} using u_{j2} and v_{j2} in recurrence relations in (16)

$$u_{j3}(t) = -(2 - \frac{sin(h)}{h})^3 sin(jh)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$
$$v_{j3}(t) = (2 - \frac{sin(h)}{h})^3 cos(jh)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$

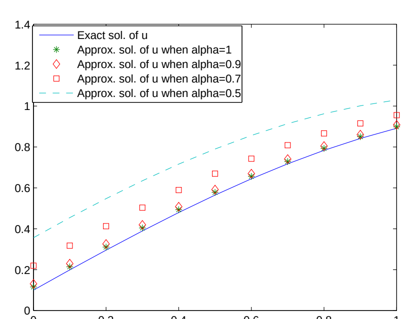
In general we can find u_{jn} and v_{jn} using u_{jn-1} and v_{jn-1} in recurrence relations in (16).Summing all above approximations, we have

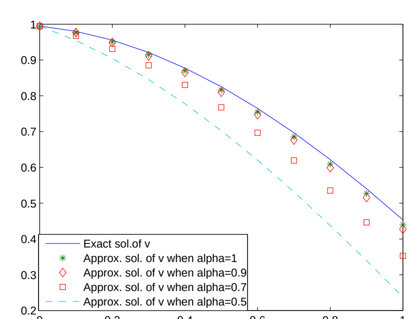
$$\begin{aligned} u_j(t) = sin(jh)&\left[1 - \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{4\alpha}}{\Gamma(4\alpha+1)} - ...\right] + \\ cos(jh)&\left[\frac{t^{\alpha}}{\Gamma(\alpha+1)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{t^{5\alpha}}{\Gamma(5\alpha+1)} - ...\right] \\ v_j(t) = cos(jh)&\left[1 - \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{4\alpha}}{\Gamma(4\alpha+1)} - ...\right] + \\ sin(jh)&\left[\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{t^{5\alpha}}{\Gamma(5\alpha+1)} - ...\right] \end{aligned} \tag{17}$$

If we put $\alpha = 1$ the solution in (17) reduces in compact form

$$(u_j(t), v_j(t)) = (sin((jh) + at), cos((jh) + at))$$

where $a = 2 - \frac{sinh}{h}$





Example 4.2. *Consider the system of nonlinear fractional partial differential equations as*

$$\begin{aligned} D_t^\alpha u + vu_x + u = 1 \\ D_t^\alpha v - uv_x - v = 1 \end{aligned} \tag{18}$$

with initial conditions

$$u(x,0) = e^x, \quad v(x,0) = e^{-x} \tag{19}$$

It is called initial value problem (IVP). The system (18) has wide applications in evolution models, the shallow water waves[14, 15, 24]. The discrete form of IVP (18)-(19) is

$$\begin{aligned} D_t^\alpha u_j(t) + v_j(t) D_h u_j(t) + u_j(t) = 1 \\ D_t^\alpha v_j(t) - u_j(t) D_h v_j(t) - v_j(t) = 1 \end{aligned} \tag{20}$$

with initial conditions

$$u_{j0} = e^{jh}, \quad v_{j0} = e^{-jh} \tag{21}$$

Operating the operator J^α on equations in(20)and using initial conditions, we have

$$\begin{aligned} u_j(t) = e^{jh} + \frac{t^\alpha}{\Gamma(\alpha+1)} - J^\alpha(v_j(t) D_h u_j(t) + u_j(t)) \\ v_j(t) = e^{-jh} + \frac{t^\alpha}{\Gamma(\alpha+1)} - J^\alpha(u_j(t) D_h v_j(t) + v_j(t)) \end{aligned} \tag{22}$$

Applying Adomian procedure and assume equations in (22) have series solution

$$u_j(t) = \sum_{n=0}^{\infty} u_{jn}(t), \quad v_j(t) = \sum_{n=0}^{\infty} v_{jn}(t) \tag{23}$$

and the nonlinear operators in equations (22) are defined as

$$v_j(t) D_h u_j(t) = \sum_{n=0}^{\infty} A_n, \quad u_j(t) D_h v_j(t) = \sum_{n=0}^{\infty} B_n \tag{24}$$

where A_n and B_n are the Adomian polynomials, which can be generated for any form of nonlinearity. Substituting equations (23) and (24) in equations (22) which yield

$$\begin{aligned} \sum_{n=0}^{\infty} u_{jn}(t) = e^{jh} + \frac{t^\alpha}{\Gamma(\alpha+1)} - J^\alpha\Big(\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} u_{jn}(t)\Big) \\ \sum_{n=0}^{\infty} v_{jn}(t) = e^{jh} + \frac{t^\alpha}{\Gamma(\alpha+1)} - J^\alpha\Big(\sum_{n=0}^{\infty} B_n + \sum_{n=0}^{\infty} v_{jn}(t)\Big) \end{aligned} \tag{25}$$

Therefore the recursive relations are

$$\begin{aligned} u_{j0} &= e^{jh} \\ u_{jn+1}(t) &= -J^\alpha\Big(\sum_{n=0}^{\infty} A_n + u_{jn}(t)\Big) \end{aligned} \tag{26}$$

and

$$\begin{aligned} v_{j0} &= e^{-jh} \\ v_{jn+1}(t) &= -J^\alpha\Big(\sum_{n=0}^{\infty} B_n + v_{jn}(t)\Big) \end{aligned} \tag{27}$$

As stated before, the Adomian polynomials can be constructed as follow. The first few Adomian polynomials for given nonlinear term are given below.

$$A_0 = v_{j0} D_h u_{j0}$$
$$A_1 = v_{j1}(t) D_h u_{j0} + v_{j0} D_h u_{j1}(t)$$
$$A_2 = v_{j2}(t) D_h u_{j0} + v_{j1}(t) D_h u_{j1}(t) + v_{j0} D_h u_{j2}(t)$$
$$A_3 = v_{j3}(t) D_h u_{j0} + v_{j2}(t) D_h u_{j1}(t) + v_{j1}(t) D_h u_{j2}(t) + v_{j0} D_h u_{j3}(t)$$

and so on.
Similarly Adomian polynomials for B_n are given as

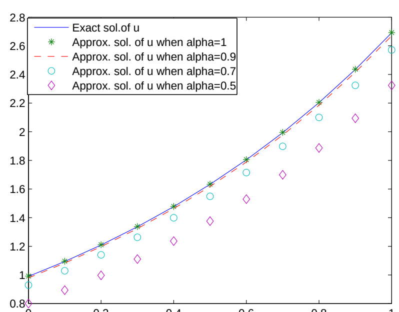
$$B_0 = u_{j0} D_h v_{j0}$$
$$B_1 = u_{j1}(t) D_h v_{j0} + u_{j0} D_h v_{j1}(t)$$
$$B_2 = u_{j2}(t) D_h v_{j0} + u_{j1}(t) D_h v_{j1}(t) + u_{j0} D_h v_{j2}(t)$$
$$B_3 = u_{j3}(t) D_h v_{j0} + u_{j2}(t) D_h v_{j1}(t) + u_{j1}(t) D_h v_{j2}(t) + u_{j0} D_h v_{j3}(t)$$

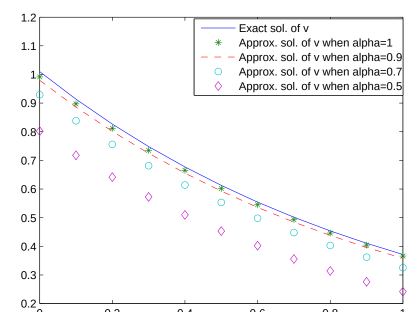
and so on.
By using recursive relations we find the approximations $u_{j1}, u_{j2}, ...$ and $v_{j1}, v_{j2}, ...$

$$u_{j0} = e^{jh}, \quad v_{j0} = e^{-jh}$$
$$u_{j1}(t) = (1 - \frac{sin}{h} + e^{jh}) \frac{t^\alpha}{\Gamma(\alpha+1)}$$
$$v_{j1}(t) = (1 - \frac{sin}{h} + e^{-jh}) \frac{t^\alpha}{\Gamma(\alpha+1)}$$

$$u_{j2}(t) = \left[(\frac{sin}{h})^2 \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - (\frac{sin}{h}) \frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \right] e^{jh} + (\frac{sin}{h}) \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{\alpha+1}}{\Gamma(\alpha+2)}$$
$$v_{j2}(t) = \left[(\frac{sin}{h})^2 \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - (\frac{sin}{h}) \frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \right] e^{-jh} - (\frac{sin}{h}) \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{\alpha+1}}{\Gamma(\alpha+2)}$$

and so on.





Example 4.3. *Consider the following nonlinear fractional order system*

$$\begin{aligned} D_t^\alpha u + v_x w_y - v_y w_x &= -u \\ D_t^\alpha v + w_x u_y + w_y u_x &= v \\ D_t^\alpha w + u_x v_y + u_y v_x &= w \end{aligned} \tag{28}$$

initial conditions

$$u(x,y,0) = e^{x+y}, \quad v(x,y,0) = e^{x-y}, \quad w(x,y,0) = e^{-x+y} \tag{29}$$

is called initial value problem (IVP) for nonlinear system. The discrete form of IVP (28)-(29) is

$$\begin{aligned} D_t^\alpha u_{i,j}(t) + D_h v_{i,j}(t) D_k w_{i,j}(t) - D_k v_{i,j}(t) D_h w_{i,j}(t) &= -u_{i,j}(t) \\ D_t^\alpha v_{i,j}(t) + D_h w_{i,j}(t) D_k u_{i,j}(t) + D_k w_{i,j}(t) D_h u_{i,j}(t) &= v_{i,j}(t) \\ D_t^\alpha w_{i,j}(t) + D_h u_{i,j}(t) D_k v_{i,j}(t) + D_k u_{i,j}(t) D_h v_{i,j}(t) &= w_{i,j}(t) \end{aligned} \tag{30}$$

with initial conditions

$$u_{i,j0} = e^{ih+jk}, \quad v_{i,j0} = e^{ih-jk}, \quad w_{i,j0} = e^{-ih+jk} \tag{31}$$

is called discrete IVP for system (28)-(29). The standard central differences are defined as

$$D_h u_{i,j}(t) = \frac{u_{i+1,j}(t) - u_{i-1,j}(t)}{2h}, \quad D_k u_{i,j}(t) = \frac{u_{i,j+1}(t) - u_{i,j-1}(t)}{2k}$$

Similarly for $D_h v_{i,j}(t), D_h w_{i,j}(t), D_k v_{i,j}(t) \& D_k w_{i,j}(t)$. Operating the operator J^α on both sides of equations in (30), we get

$$\begin{aligned} u_{i,j}(t) &= e^{ih+jk} - J^\alpha\Big(u_{i,j}(t) + D_h v_{i,j} D_k w_{i,j}(t) - D_k v_{i,j} D_h w_{i,j}(t)\Big) \\ v_{i,j}(t) &= e^{ih-jk} - J^\alpha\Big(v_{i,j}(t) - D_h w_{i,j} D_k u_{i,j}(t) - D_k w_{i,j} D_h u_{i,j}(t)\Big) \\ w_{i,j}(t) &= e^{-ih+jk} - J^\alpha\Big(w_{i,j}(t) - D_h u_{i,j} D_k v_{i,j}(t) - D_k u_{i,j} D_h v_{i,j}(t)\Big) \end{aligned} \tag{32}$$

Using the discrete ADM we assume that it has series solution

$$\begin{aligned} \sum_{n=0}^{\infty} u_{i,jn}(t) &= e^{ih+jk} - J^\alpha\Big(\sum_{n=0}^{\infty} u_{i,jn}(t) + \sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} \bar{A_n}\Big) \\ \sum_{n=0}^{\infty} v_{i,jn}(t) &= e^{ih-jk} - J^\alpha\Big(\sum_{n=0}^{\infty} v_{i,jn}(t) - \sum_{n=0}^{\infty} B_n - \sum_{n=0}^{\infty} \bar{B_n}\Big) \\ \sum_{n=0}^{\infty} w_{i,jn}(t) &= e^{-ih+jk} - J^\alpha\Big(\sum_{n=0}^{\infty} w_{i,jn}(t) + \sum_{n=0}^{\infty} C_n - \sum_{n=0}^{\infty} \bar{C_n}\Big) \end{aligned} \tag{33}$$

where $A_n, \bar{A_n}, B_n, \bar{B_n}, C_n$ and $\bar{C_n}$ are Adomian polynomials that represents the nonlinear terms. These few polynomials can be formed for each nonlinear terms. Here we list few terms of Adomian polynomials.For $D_h v_{i,j}(t) D_k w_{i,j}(t)$, we get

$$\begin{aligned} A_0 &= D_h v_{i,j0} D_k w_{i,j0} \\ A_1 &= D_h v_{i,j1}(t) D_k w_{i,j0} + D_h v_{i,j0} D_k w_{i,j1}(t) \\ A_2 &= D_h v_{i,j2}(t) D_k w_{i,j0} + D_h v_{i,j1}(t) D_k w_{i,j1}(t) + D_h v_{i,j0} D_k w_{i,j2}(t) \end{aligned}$$

and for $D_h w_{i,j} D_k v_{i,j}(t)$, we have

$$\begin{aligned}
\bar{A_0} &= D_k v_{i,j0} D_h w_{i,j0} \\
\bar{A_1} &= D_k v_{i,j1}(t) D_h w_{i,j0} + D_k v_{i,j0} D_h w_{i,j1}(t) \\
\bar{A_2} &= D_k v_{i,j2}(t) D_h w_{i,j0} + D_k v_{i,j1}(t) D_h w_{i,j1}(t) + D_k v_{i,j0} D_h w_{i,j2}(t)
\end{aligned}$$

Similarly, for $B_n, \bar{B_n}, C_n$ and $\bar{C_n}$, we can find terms. Using these polynomials and by employing the appropriate recursive relations we find

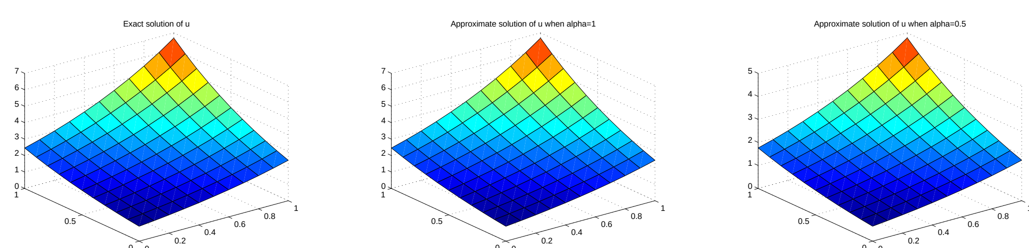
$$\begin{aligned}
\Big(u_{i,j0}, v_{i,j0}, w_{i,j0}\Big) &= \Big(e^{ih+jk}, e^{ih-jk}, e^{-ih+jk}\Big) \\
\Big(u_{i,j1}(t), v_{i,j1}(t), w_{i,j1}(t)\Big) &= \left(\frac{-e^{ih+jk}t^{\alpha}}{\Gamma(\alpha+1)}, \frac{e^{ih-jk}t^{\alpha}}{\Gamma(\alpha+1)}, \frac{e^{-ih+jk}t^{\alpha}}{\Gamma(\alpha+1)}\right) \\
\Big(u_{i,j2}(t), v_{i,j2}(t), w_{i,j2}(t)\Big) &= \left(\frac{e^{ih+jk}t^{2\alpha}}{\Gamma(2\alpha+1)}, \frac{e^{ih-jk}t^{2\alpha}}{\Gamma(2\alpha+1)}, \frac{e^{-ih+jk}t^{2\alpha}}{\Gamma(2\alpha+1)}\right) \\
\Big(u_{i,j3}(t), v_{i,j3}(t), w_{i,j3}(t)\Big) &= \left(\frac{-e^{ih+jk}t^{3\alpha}}{\Gamma(3\alpha+1)}, \frac{e^{ih-jk}t^{3\alpha}}{\Gamma(3\alpha+1)}, \frac{e^{-ih+jk}t^{3\alpha}}{\Gamma(3\alpha+1)}\right)
\end{aligned}$$

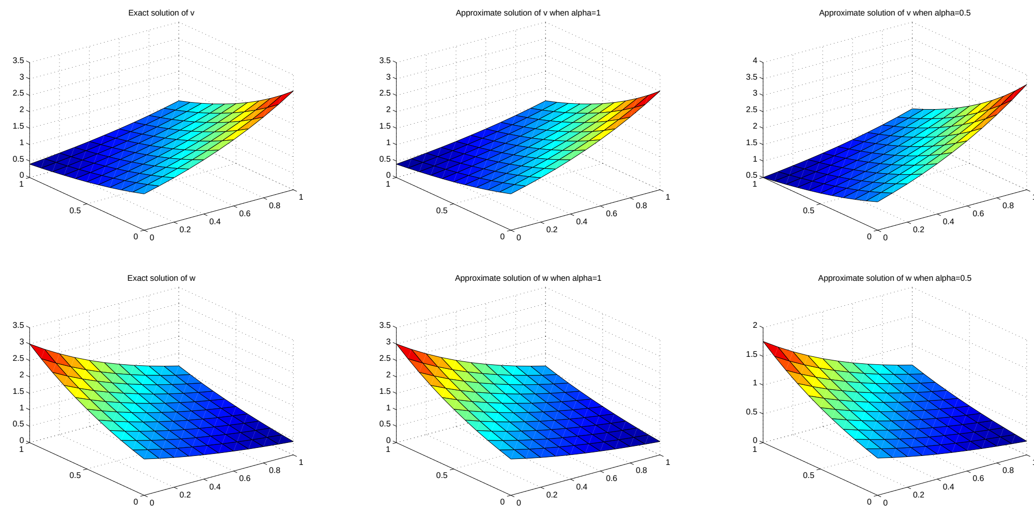
and so on.

$$\begin{aligned}
u_{i,j}(t) &= e^{ih+jk} - \frac{e^{ih+jk}t^{\alpha}}{\Gamma(\alpha+1)} + \frac{e^{ih+jk}t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{e^{ih+jk}t^{3\alpha}}{\Gamma(3\alpha+1)} + ... \\
&= e^{ih+jk}\left(1 - \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + ...\right) \\
v_{i,j}(t) &= e^{ih-jk} + \frac{e^{ih-jk}t^{\alpha}}{\Gamma(\alpha+1)} + \frac{e^{ih-jk}t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{e^{ih-jk}t^{3\alpha}}{\Gamma(3\alpha+1)} + ... \\
&= e^{ih-jk}\left(1 + \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + ...\right) \\
w_{i,j}(t) &= e^{-ih+jk} + \frac{e^{-ih+jk}t^{\alpha}}{\Gamma(\alpha+1)} + \frac{e^{-ih+jk}t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{e^{-ih+jk}t^{3\alpha}}{\Gamma(3\alpha+1)} + ... \\
&= e^{-ih+jk}\left(1 - \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + ...\right)
\end{aligned}$$

If we put $\alpha = 1$ in above equations, we get the compact solution as follows

$$\Big(u_{i,j}(t), v_{i,j}(t), w_{i,j}(t)\Big) = \Big(e^{-ih+jk-t}, e^{ih-jk+t}, e^{-ih+jk+t}\Big)$$





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